More on the guessing technique for the damped harmonic oscillator

Last class we guessed that solutions of the form $y(t) = e^{\lambda t}$ might be solutions to the damped harmonic oscillator

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0.$$

In fact, we saw that $y(t) = e^{\lambda t}$ is a solution if and only if

$$m\lambda^2 + b\lambda + k = 0.$$

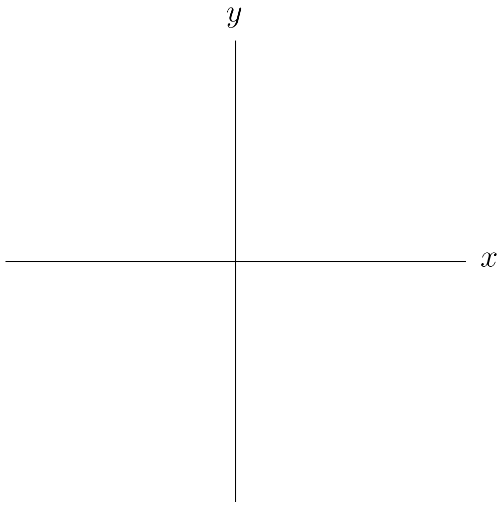
This equation is called the *characteristic equation* of the harmonic oscillator, and the polynomial $p(\lambda) = m\lambda^2 + b\lambda + k$ is its *characteristic polynomial*.

Example. Consider the harmonic oscillator

$$\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2x = 0.$$

Its characteristic equation is

Let's plot these solutions with `HPGSystemSolver`. What are the corresponding solution curves and component graphs?



Euler's method for a system

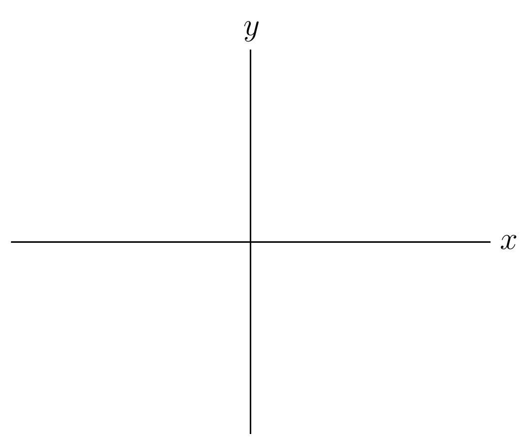
We can use the vector field for a system to produce numerical approximations for the solutions.

Example. Consider the IVP

$$\frac{dx}{dt} = -y \qquad (x_0, y_0) = (2, 0).$$
$$\frac{dy}{dt} = x - y$$

The `EulersMethodForSystems` tool demonstrates the method. We pick a large step size $\Delta t = 0.5$ so that we can see the method in action.

k	x_k	y_k	m_k	n_k
0	2	0		
1				
2				
3				
4				
5				
6				



Now let's derive the general equations for Euler's method for an autonomous IVP of the form

$$\begin{aligned} \frac{dx}{dt} &= f(x, y) \\ \frac{dy}{dt} &= g(x, y) \end{aligned} \qquad (x(t_0), y(t_0)) = (x_0, y_0).$$

Euler's method for systems is just as easy to program as Euler's method for equations. Once again here's how we can program it with a spreadsheet.

	A	B	C	D	E	F	G
0	0	2	0	0.5			
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							

There are two spreadsheets posted on the course web site—one for the example above and one for the following example.

Example. Consider the predator-prey system

$$\frac{dR}{dt} = R - 0.2RF$$
$$\frac{dF}{dt} = -0.3F + 0.1RF$$

along with the initial condition $(R_0, F_0) = (1, 2)$. Using the spreadsheet on the web site, we see that Euler's method has trouble approximating periodic solutions.

`HPGSystemSolver` uses a more sophisticated fixed-step-size algorithm called the Runge-Kutta method. It usually works better than Euler's method, but there are equations for which any fixed-step-size algorithm is not appropriate.