General 2D first-order autonomous systems

In general, a 2D first-order autonomous system of ordinary differential equations has

- one independent variable and
- two dependent variables.
- The independent variable does not appear on the right-hand sides of the differential equations.

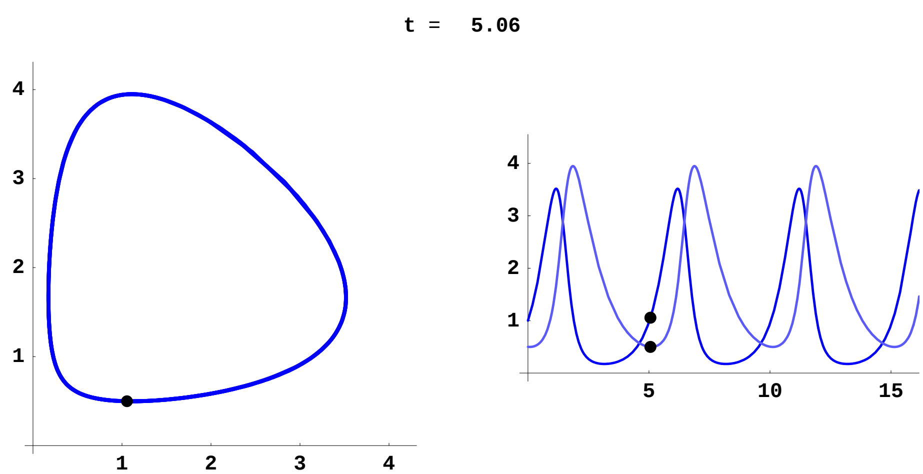
Example. Recall the predator-prey systems we discussed briefly at the start of the semester

$$\frac{dR}{dt} = aR - bRF$$
$$\frac{dF}{dt} = -cF + dRF.$$

Let's go through some terminology:

- initial condition:

- solution to an initial-value problem:



See the web site for the entire animation and for a related 3D-animation.

- equilibrium solutions:

- component graphs:

- phase plane:

- solution curve in the phase plane:

- phase portrait:

One skill that you will learn is how to make a rough sketch of the component graphs from the solution curve. There is a tool on your CD called `DESketchPad` which will help you practice.

General second-order autonomous equations

In general, a second-order autonomous equation has

- one independent variable and
- one dependent variable.

It has the form

$$\frac{d^2y}{dt^2} = f\left(y, \frac{dy}{dt}\right).$$

Example. Simple mass-spring system

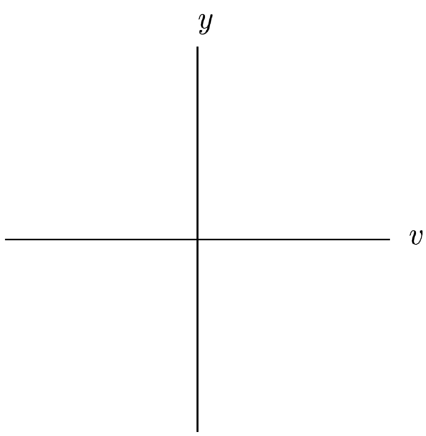
Hooke's Law: The restoring force of the spring is proportional to the displacement from its rest position.

Using Newton's law $F = ma$, we get

In what ways is the mass-spring system similar to the predator-prey system?

An initial condition for the predator-prey system is a pair (R_0, F_0) of population values.

An initial condition for the mass-spring system is also a pair (y_0, v_0). The first number indicates the initial displacement and the second number indicates the initial velocity.



Let's consider the special case where $k = m$. We get

$$\frac{d^2y}{dt^2} = -y,$$

and we can guess some solutions to this equation:

We can perform a mathematical reduction to the second-order equation for the mass-spring system so that it resembles the predator-prey system: