Building phase lines:

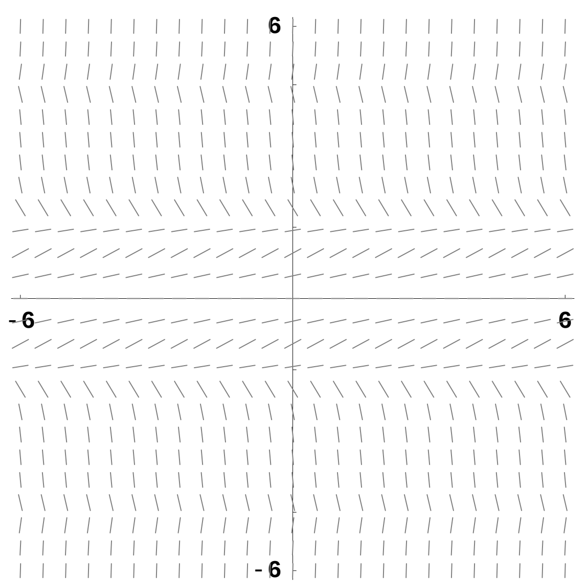
Given an autonomous equation $dy/dt = f(y)$, you need to know how to build the phase line from the formula for $f(y)$ and from the graph of $f(y)$.

Example 1. $\dfrac{dy}{dt} = y^2 \cos y$

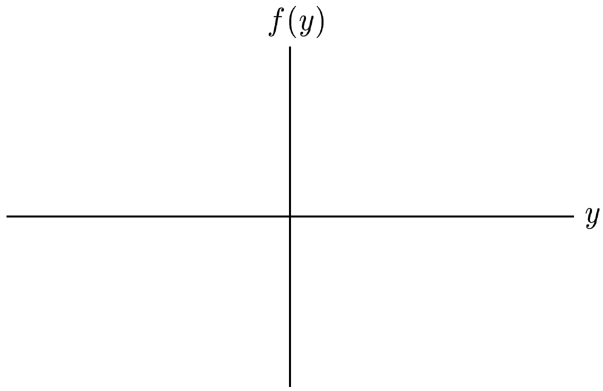
At the end of class on Friday, we made two observations:

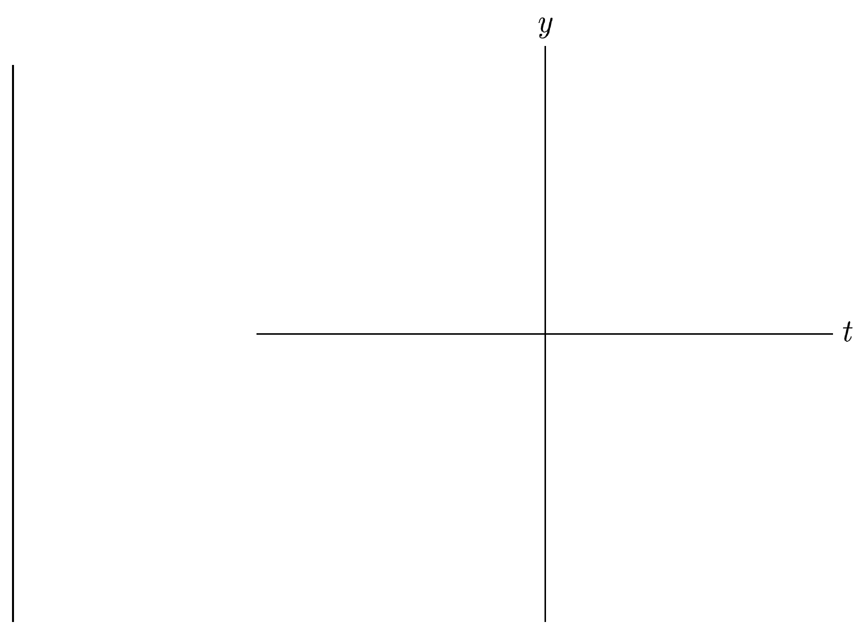
1. The equilibrium points are $y = 0$ and $y = \pm\pi/2, \pm 3\pi/2, \pm 5\pi/2, \ldots$
2. $\text{sign}(dy/dt) = \text{sign}(\cos y)$

Now we can build the phase line and get a qualitative idea of what the solutions do.



Example 2. $\dfrac{dy}{dt} = f(y)$ where $f(y)$ is given by the graph





Parameters, Qualitative Equivalence, and Bifurcations

Let's return to the logistic model of population growth

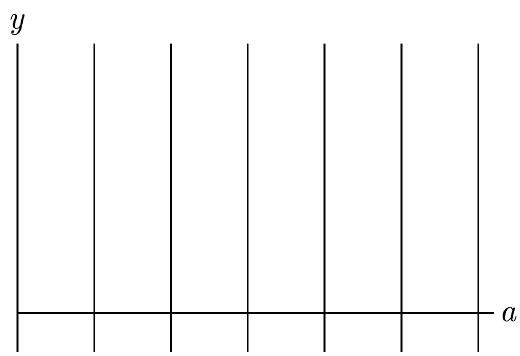
$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right)$$

and modify this model to account for constant harvesting:

Example. $\dfrac{dy}{dt} = y(1-y) - a$

There is a tool in `DETools` called `PhaseLines`, and it helps us analyze phase lines and various graphs as we vary certain parameters (the parameter a in this case).

We can summarize the behavior of this one-parameter family of differential equations using the bifurcation diagram.



Now let's sketch and interpret the bifurcation diagram for the logistic population model with constant harvesting

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{N}\right) - C.$$