

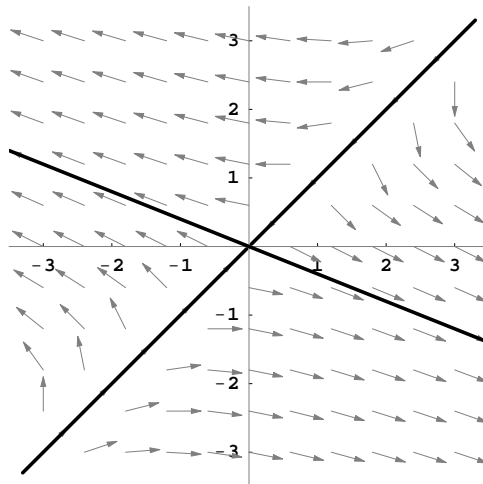
Example. Last class we found the general solution to

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix} \mathbf{Y}$$

It is

$$\mathbf{Y}(t) = k_1 e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + k_2 e^{6t} \begin{pmatrix} 5 \\ -2 \end{pmatrix}.$$

Using `HPGSystemSolver`, we plot the phase portrait for this system.



Facts about eigenvalues and eigenvectors: Given a 2×2 matrix \mathbf{A} ,

1. The characteristic equation can have two real roots, one real root of multiplicity two, or two complex conjugate roots.
2. Given an eigenvector \mathbf{Y}_0 associated to an eigenvalue λ , then any nonzero scalar multiple \mathbf{Y}_0 is also an eigenvector associated to λ .
3. Eigenvectors associated to distinct eigenvalues are linearly independent.

Summary of Case of Two Distinct Real Eigenvalues

Suppose \mathbf{A} is a matrix with two eigenvalues λ_1 and λ_2 . To be consistent, we will assume that $\lambda_1 < \lambda_2$, that \mathbf{V}_1 is an eigenvector associated to λ_1 , and that \mathbf{V}_2 is an eigenvector associated to λ_2 . The general solution of

$$\frac{d\mathbf{Y}}{dt} = \mathbf{A}\mathbf{Y}$$

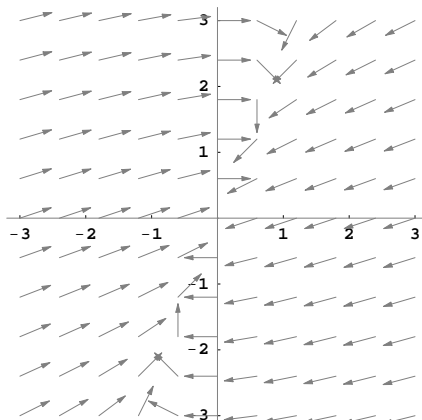
is

$$\mathbf{Y}(t) = k_1 e^{\lambda_1 t} \mathbf{V}_1 + k_2 e^{\lambda_2 t} \mathbf{V}_2.$$

Case 1: $\lambda_1 < \lambda_2 < 0$.

Example. Consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{Y}.$$



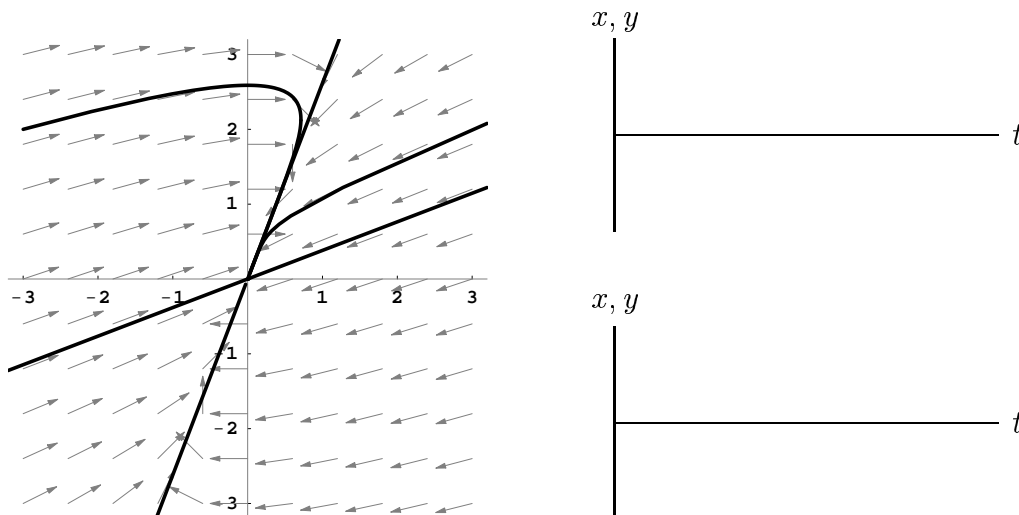
Sketching component graphs

Once we understand the phase portrait, we should also be able to sketch the component graphs without HPGSystemSolver.

For example, once again consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} -3 & 1 \\ -1 & 0 \end{pmatrix} \mathbf{Y}.$$

Let's sketch the $x(t)$ - and $y(t)$ -graphs that correspond to the initial conditions $(-3, 2)$ and $(3, 2)$.

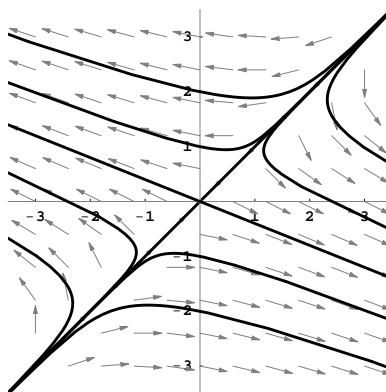


Case 2: $\lambda_1 < 0 < \lambda_2$.

We did this case at the start of class.

Example. Consider

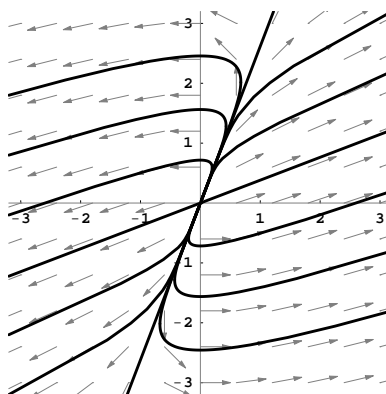
$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 4 & -5 \\ -2 & 1 \end{pmatrix} \mathbf{Y}.$$



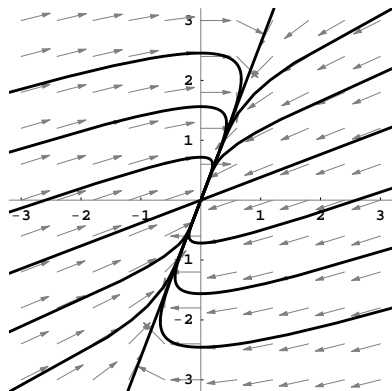
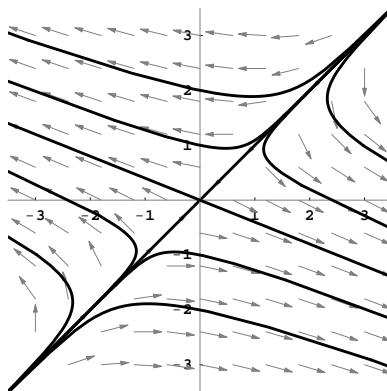
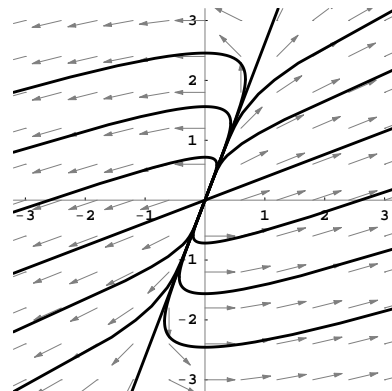
Case 3: $0 < \lambda_1 < \lambda_2$.

Example. Consider

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 3 & -1 \\ 1 & 0 \end{pmatrix} \mathbf{Y}.$$



Summary for real and distinct eigenvalues

sink ($\lambda_1 < \lambda_2 < 0$)saddle ($\lambda_1 < 0 < \lambda_2$)source ($0 < \lambda_1 < \lambda_2$)