

## Sinusoidal forcing

Today we are going to study forced equations where the forcing function is sinusoidal (either sine or cosine). But first I want to remind you of the calculation that we have done at least twice before. It saves a little time in these “guessing” problems.

Time saver: Consider the function  $y_p(t) = e^{\lambda t}$  and calculate

$$m \frac{d^2 y_p}{dt^2} + b \frac{dy_p}{dt} + k y_p.$$

Now let’s apply this guessing technique to sinusoidally forced linear equations.

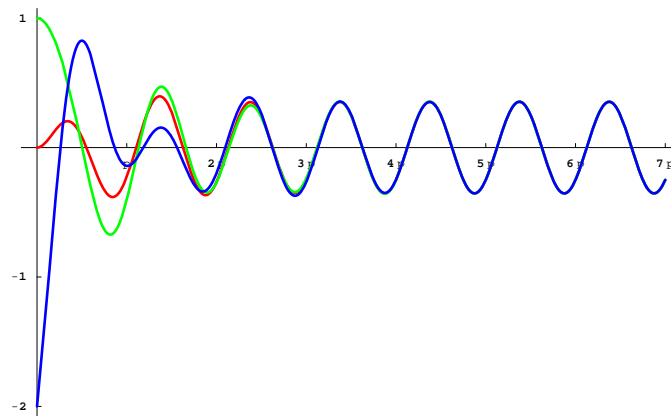
**Example.** Let’s calculate the general solution to the equation

$$\frac{d^2 y}{dt^2} + \frac{dy}{dt} + 2y = \cos 2t.$$

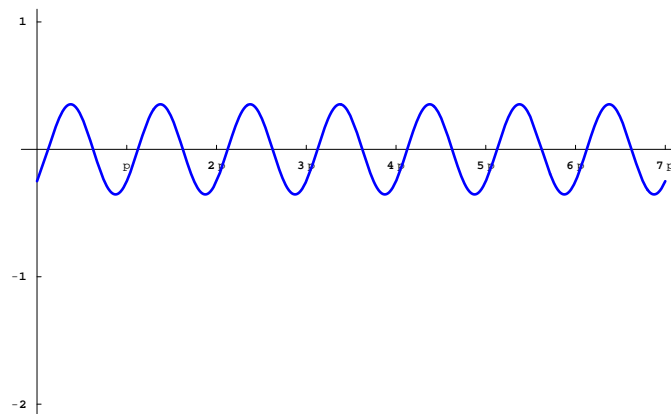


We can see the implications of this computation by entering this equation into `ForcedMassSpring` on the CD.

Here are the graphs of three solutions:



Here is the graph of the steady-state solution:



A little translation:

Consider the second-order linear forced equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = f(t)$$

where  $m$ ,  $b$ , and  $k$  are all positive.

Engineering terminology:

**forced response**—any solution to the forced equation.

**steady-state response**—behavior of the forced response over the long term.

**natural (or free) response**—any solution of the associated homogeneous equation.

Why are initial conditions essentially irrelevant?

When we guess a solution of the form  $y_p(t) = ae^{i\omega t}$  and compute the complex number  $a$ , we have essentially determined everything we need to know about the steady-state solution. Euler's formula gives us a nice way of determining the amplitude, frequency, and phase angle of the steady-state solution immediately from  $a$ :

