

More on the guessing technique for the damped harmonic oscillator

**Example.** Last class we applied a guessing technique to produce two solutions for the harmonic oscillator equation

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0.$$

Using the characteristic equation, we obtained

$$y_1(t) = e^{-2t} \quad \text{and} \quad y_2(t) = e^{-t}.$$

The corresponding velocity functions are

$$v_1(t) = -2e^{-2t} \quad \text{and} \quad v_2(t) = -e^{-t}.$$

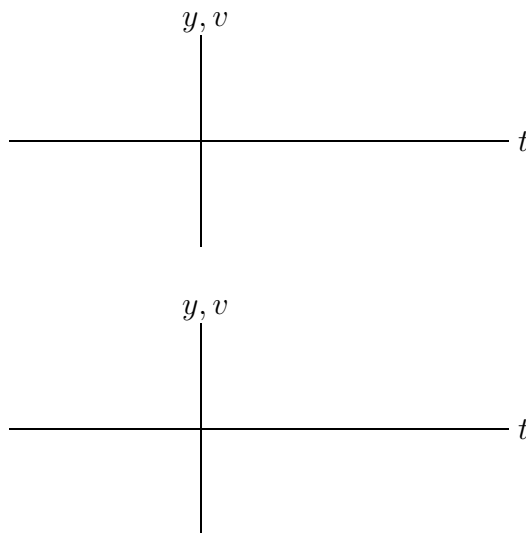
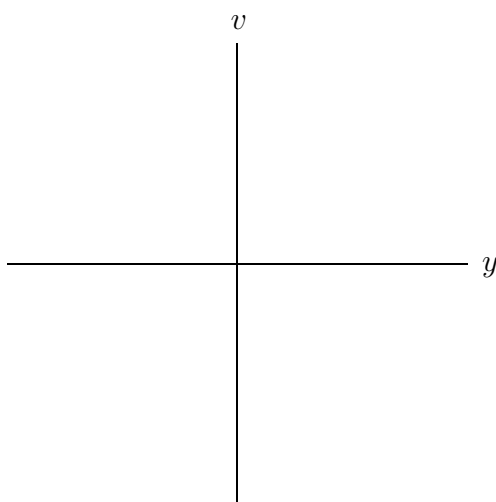
In vector form, these solutions are written as

$$\mathbf{Y}_1(t) = \begin{pmatrix} e^{-2t} \\ e^{-t} \end{pmatrix} = e^{-2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

and

$$\mathbf{Y}_2(t) = \begin{pmatrix} e^{-t} \\ -e^{-t} \end{pmatrix} = e^{-t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

Let's plot these solutions with `HPGSystemSolver`. What are the corresponding solution curves and component graphs?



Euler's method for a system

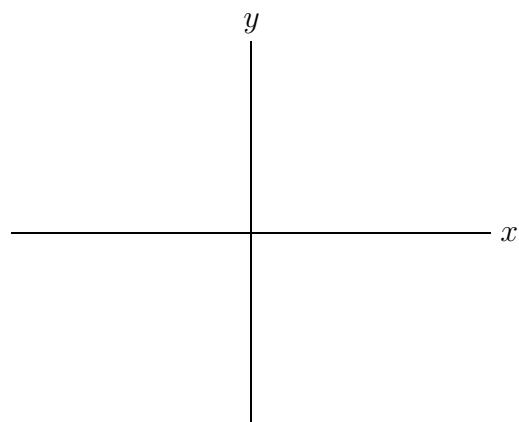
We can use the vector field for a system to produce numerical approximations for the solutions.

**Example.** Consider the IVP

$$\begin{aligned} \frac{dx}{dt} &= -y \\ \frac{dy}{dt} &= x - y \end{aligned} \quad (x_0, y_0) = (2, 0).$$

The `EulersMethodForSystems` tool demonstrates the method. We pick a large step size  $\Delta t = 0.5$  so that we can see the method in action.

$k$	$x_k$	$y_k$	$m_k$	$n_k$
0	2	0		
1				
2				
3				
4				
5				
6				



Now let's derive the general equations for Euler's method for an autonomous IVP of the form

$$\begin{aligned} \frac{dx}{dt} &= f(x, y) \\ \frac{dy}{dt} &= g(x, y) \end{aligned} \quad (x(t_0), y(t_0)) = (x_0, y_0).$$

Euler's method for systems is just as easy to program as Euler's method for equations. Once again here's how we can program it with a spreadsheet.

	A	B	C	D	E	F	G
0	0	2	0	0.5			
1							
2							
3							
4							
5							
6							
7							
8							
9							
10							
11							
12							
13							
14							
15							
16							
17							
18							

There are two spreadsheets posted on the course web site—one for the example above and one for the following example.

**Example.** Consider the predator-prey system

$$\begin{aligned}\frac{dR}{dt} &= R - 0.2RF \\ \frac{dF}{dt} &= -0.3F + 0.1RF\end{aligned}$$

along with the initial condition  $(R_0, F_0) = (1, 2)$ . Using the spreadsheet on the web site, we see that Euler's method has trouble approximating periodic solutions.

`HPGSystemSolver` uses a more sophisticated fixed-step-size algorithm called the Runge-Kutta method. It usually works better than Euler's method, but there are equations for which any fixed-step-size algorithm is not appropriate.