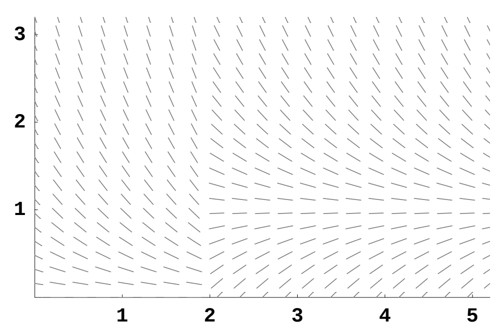
Finish initial-value problem from last class

Last class we got half way through solving an initial-value problem with discontinuous forcing. I want to finish that example today.

Example. Solve the initial-value problem $\dfrac{dv}{dt} + v = u_2(t), \quad v(0) = 3.$

1. Transform both sides of the equation: $s\mathcal{L}[v] - 3 + \mathcal{L}[v] = \dfrac{e^{-2s}}{s}$
2. Solve for $\mathcal{L}[v]$: $\mathcal{L}[v] = \dfrac{3}{s+1} + \dfrac{e^{-2s}}{s(s+1)}$
3. Calculate the inverse Laplace transform:

Now let's plot this solution using `HPGSolver`.



Laplace transforms and second-order equations

So far we have only applied the Laplace transform to first-order equations. Now we consider second-order equations.

Recall the rule for Laplace transforms of derivatives: $\mathcal{L}\left[\frac{dy}{dt}\right] = s\mathcal{L}[y] - y(0)$

What does this rule say about $\mathcal{L}\left[\frac{d^2y}{dt^2}\right]$?

Now that we have this rule, we also need to add to our table of Laplace transforms. Since sine and cosine often appear as parts of the solutions to second-order equations, let's determine their Laplace transforms.

There are a number of ways to compute these transforms—using integration by parts, using Euler's formula, and even using the fact that sine and cosine are solutions to certain very special second-order equations. On April 14 we saw that

$$\mathcal{L}[e^{i\omega t}] = \frac{1}{s - i\omega}.$$

Let's use this to determine $\mathcal{L}[\sin \omega t]$ and $\mathcal{L}[\cos \omega t]$.

Now that we know the transforms of sine and cosine, let's see how we use them.

Example. Compute

$$\mathcal{L}^{-1}\left[\frac{2s+1}{s^2+9}\right].$$

Now for a little practice with the third rule for transforms:

Example. Compute

$$\mathcal{L}^{-1}\left[\frac{8e^{-10s}}{(s^2+9)(s^2+1)}\right].$$