Sinusoidal forcing

Today we are going to study forced equations where the forcing function is sinusoidal (either sine or cosine). But first I want to remind you of the calculation that we did at the end of last class. It saves a little time in these "guessing" problems.

Time saver: If we guess $y(t) = ae^{\lambda t}$ where a is a constant, we get

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = a\,p(\lambda)\,e^{\lambda t}$$

where $p(\lambda) = m\lambda^2 + b\lambda + k$ is the characteristic polynomial.

Now let's apply this guessing technique to sinusoidally forced linear equations.

Example. Let's calculate the general solution to the equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = \cos 2t.$$

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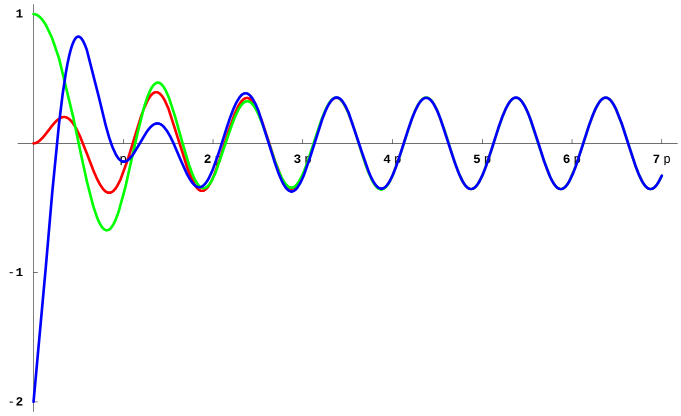
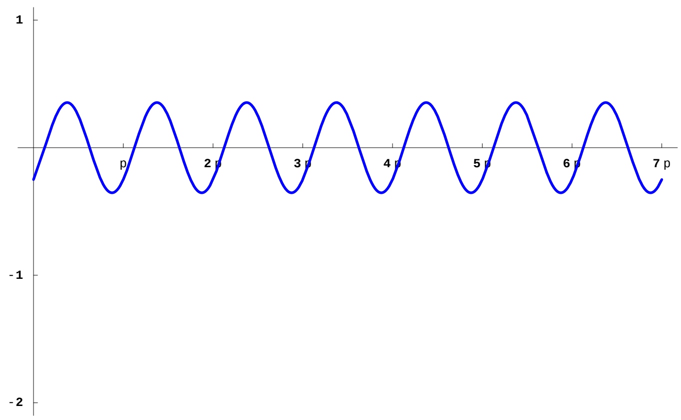
We can see the implications of this computation by entering this equation into `MassSpring` on the CD.

Here are the graphs of three solutions:



Here is the graph of the steady-state solution:



A little translation:

Consider the second-order linear forced equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = f(t)$$

where m, b, and k are all positive.

Engineering terminology:

forced response—any solution to the forced equation.

steady-state response—behavior of the forced response over the long term.

natural (or free) response—any solution of the associated homogeneous equation.

Why are initial conditions essentially irrelevant?

When we guess a solution of the form $y_p(t) = ae^{i\omega t}$ and compute the complex number a, we have essentially determined everything we need to know about the steady-state solution. Euler's formula gives us a nice way of determining the amplitude, frequency, and phase angle of the steady-state solution immediately from a: