A translation from engineering terminology to mathematical terminology

forced response—any solution to the forced equation.

steady-state response—behavior of the forced response over the long term.

natural (or free) response—any solution of the associated homogeneous equation.

Why are initial conditions essentially irrelevant?

A little more on the steady-state solution

I still owe you an explanation for why I prefer to calculate the steady-state solution using complex numbers.

On Friday, we calculated the steady-state solution

$$y_p(t) = -\tfrac{1}{4}(\cos 2t - \sin 2t)$$

for the equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 2y = \cos 2t,$$

and we did so using computations that involved complex numbers. In fact, we found $y_p(t)$ as the real part of

$$y_c(t) = -\tfrac{1}{4}(1+i)\, e^{(2i)t}.$$

The complex number

$$a = -\tfrac{1}{4}(1+i)$$

tells us everything we need to know about the steady-state solution.

In order to see why, we use polar coordinates in the complex plane (see pp. 745–747 in Appendix C of the text).

Let's rewrite $a = -\frac{1}{4}(1+i)$ in this polar form.

What does this polar representation of a tell us about the steady-state solution?

Sinusoidal forcing in the absense of damping

Now consider the mass-spring system without the dashpot.

Example. Let's find the general solution to

$$\frac{d^2y}{dt^2} + 3y = \cos \omega t.$$

Note the lack of a damping term. We want to see what happens with various forcing frequencies.

Unfortunately the parts of the solution that correspond to the associated homogeneous equation do not die out. So to get some qualitative understanding in this case, we make a simplifying assumption. We consider the solution that satisfies the initial condition $(y(0), y'(0)) = (0, 0)$.

On the web site, there is a Quicktime animation of the graphs of these solutions as we vary the forcing frequency ω. We can also visualize these solutions using a parameter in `HPGSystemSolver`.