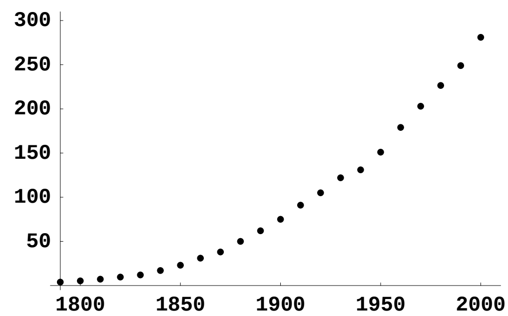
Modeling the US Population:

The data graphed as a function of time:



First Model: Malthusian Model

Assumption: Growth rate of the population is proportional to the population.

Variables: independent variable t for time (in years since 1790) and dependent variable p for population (in millions)

Malthusian model is

$$\frac{dp}{dt} = kp,$$

where k is a proportionality constant (a parameter).

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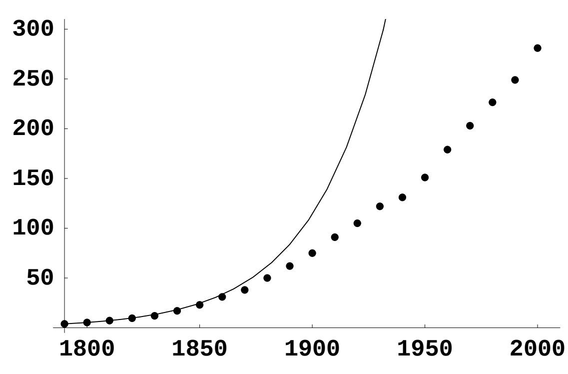
Here's the graph of $p(t)$ superimposed on the data:



Second Model: Logistic Model
Assumptions:

1. If the population is small, its growth rate is proportional to the size of the population.

2. As the population increases, its **relative growth rate** decreases.

What is a relative growth rate?

A Qualitative Analysis of the Logistic Model

We now have
$$\frac{dp}{dt} = kp\left(1 - \frac{p}{N}\right).$$

Can we determine the long-term behavior of solutions without computing the solutions first?