Modeling the US Population:

The data graphed as a function of time:

First Model: Malthusian Model
Assumption: Growth rate of the population is proportional to the population.
Variables: independent variable $t$ for time (in years since 1790) and dependent variable $p$ for population (in millions)

Malthusian model is

$$\frac{dp}{dt} = kp,$$

where $k$ is a proportionality constant (a parameter).
Here’s the graph of $p(t)$ superimposed on the data:
Second Model: Logistic Model  
Assumptions:  

1. If the population is small, its growth rate is proportional to the size of the population.  
2. As the population increases, its relative growth rate decreases.  

What is a relative growth rate?
A Qualitative Analysis of the Logistic Model

We now have

\[ \frac{dp}{dt} = kp \left( 1 - \frac{p}{N} \right). \]

Can we determine the long-term behavior of solutions without computing the solutions first?