We begin with a comment related to the example of a separable differential equation that we discussed last class.

Example. After separating variables on the differential equation
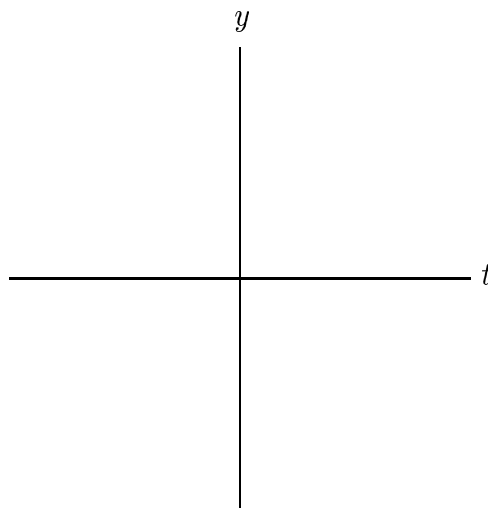
$$\frac{dy}{dt} = -2ty^2,$$

we obtained the family of solutions $y(t) = \dfrac{1}{t^2 + k}$ where k is an arbitrary constant.

For the initial condition $y(0) = \frac{1}{2}$ we have $k = 2$, and the solution is $y(t) = \dfrac{1}{t^2 + 2}$ with a domain of $-\infty < t < \infty$.

For the initial condition $y(0) = -\frac{1}{2}$ we have $k = -2$, and the solution is $y(t) = \dfrac{1}{t^2 - 2}$ with a domain of $-\sqrt{2} < t < \sqrt{2}$.

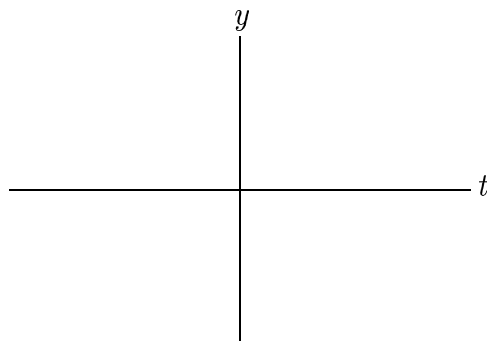We turn to `FirstOrderExamples` to get a sense of the graphs of these solutions:

What's the general solution to $\dfrac{dy}{dt} = -2ty^2$? (Think before you answer.)

Slope fields

A **slope field** in the ty-plane is a picture of a first-order differential equation
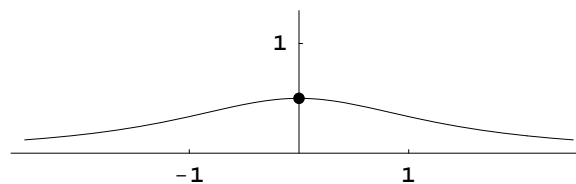
$$\frac{dy}{dt} = f(t, y).$$

The graph of a solution must be everywhere tangent to the slope field.
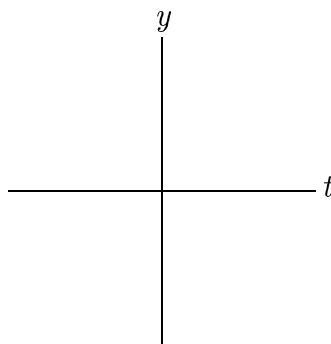
Example. Consider the differential equation

$$\frac{dy}{dt} = -2ty^2$$

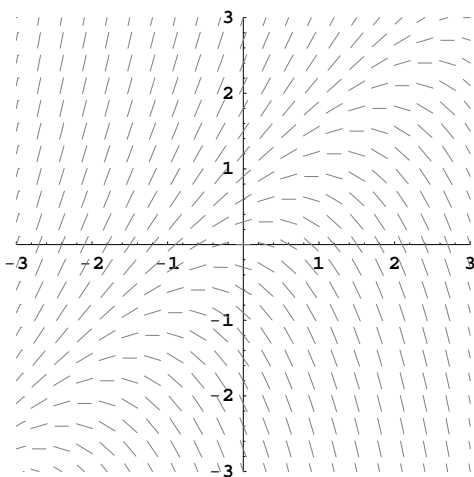on which we separated variables.

Example. Consider the differential equation
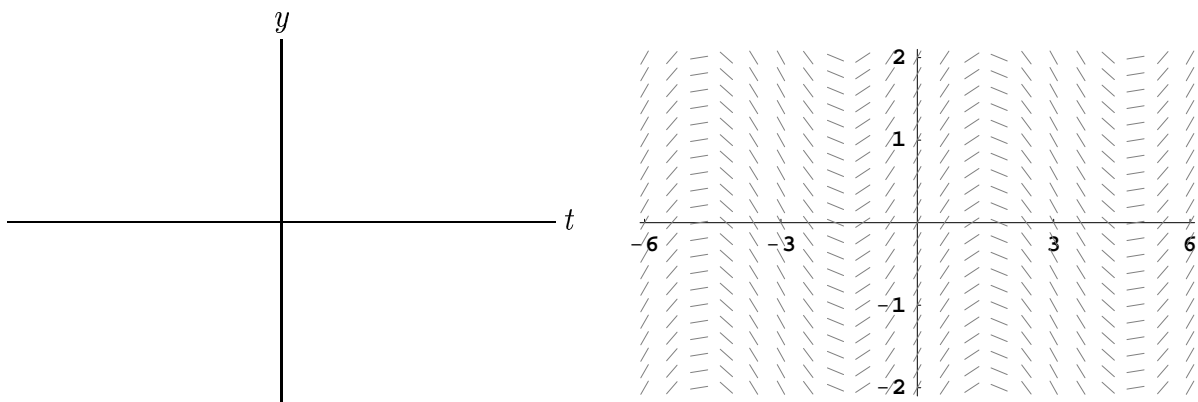
$$\frac{dy}{dt} = y - t.$$

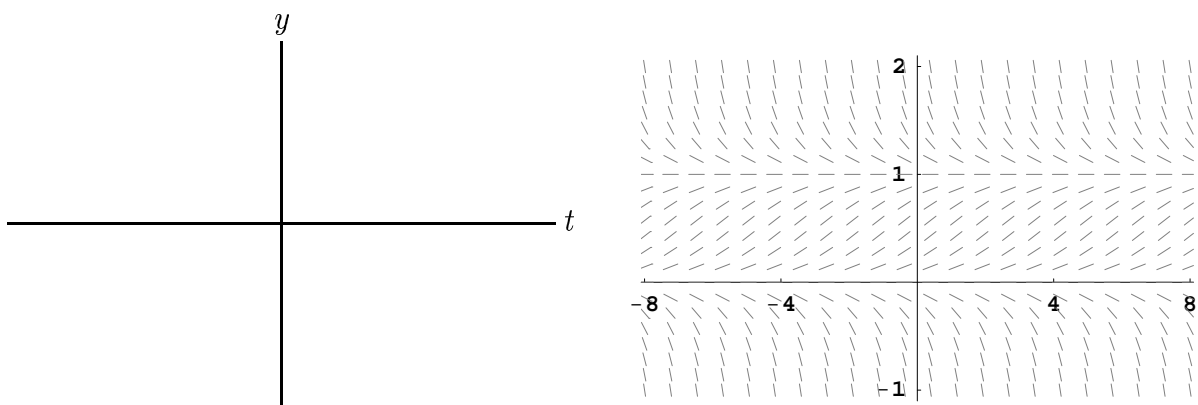Using the computer to plot the slope field, we get

The `TargetPractice` tool is a fun way to see how the slope field relates to the solutions of the differential equation.

3

Important special cases:

1. $\dfrac{dy}{dt} = f(t)$
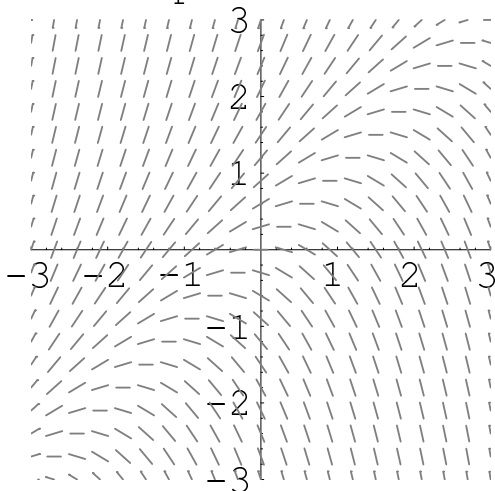
2. $\dfrac{dy}{dt} = f(y)$ (autonomous)

Typical exam problem:
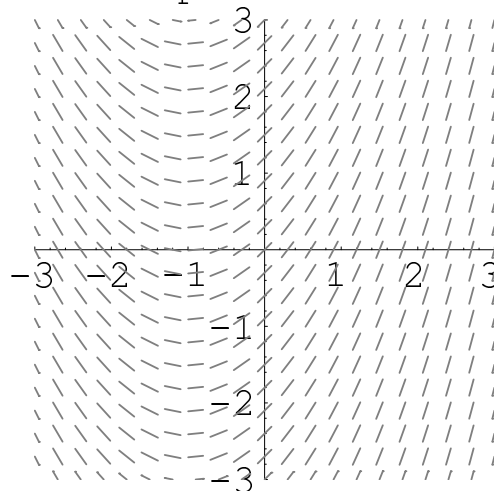
Consider the following 8 first-order equations:

1. $\dfrac{dy}{dt} = t - 1$ 2. $\dfrac{dy}{dt} = t + 1$ 3. $\dfrac{dy}{dt} = y + 1$ 4. $\dfrac{dy}{dt} = 1 - y$

5. $\dfrac{dy}{dt} = y^2 + y$ 6. $\dfrac{dy}{dt} = y(y^2 - 1)$ 7. $\dfrac{dy}{dt} = y - t$ 8. $\dfrac{dy}{dt} = y + t$

Four of the associated slope fields are shown below. Pair the slope fields with their associated equations. Provide a brief justification for your choice.
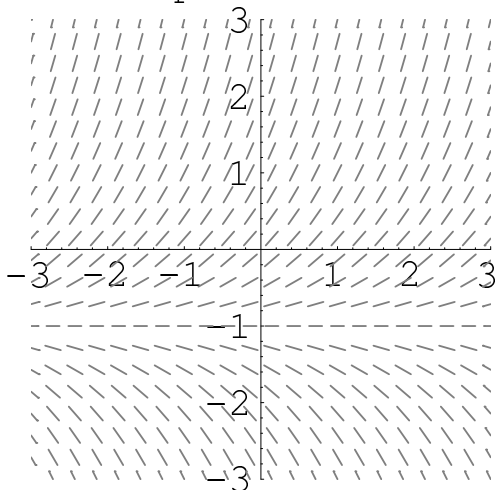

Slope Field A

Slope Field B

Slope Field C

Slope Field D