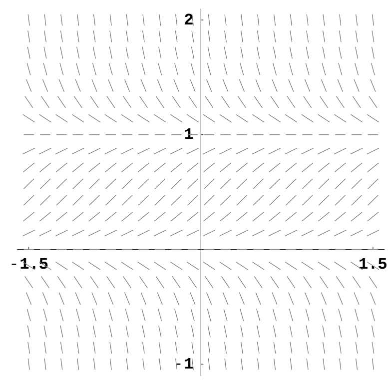
More on Uniqueness

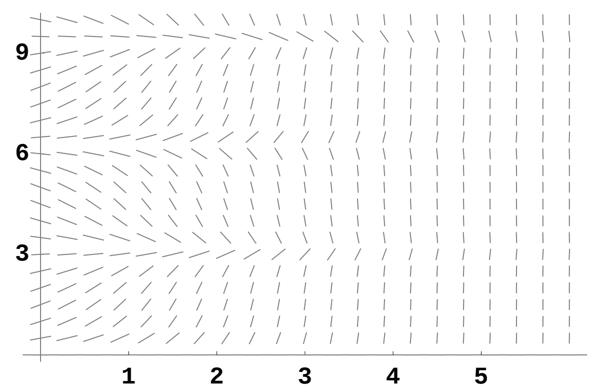
The Uniqueness Theorem has many useful consequences. Here are three examples:

Example 1. $\dfrac{dy}{dt} = -2ty^2$

Example 2. $\dfrac{dy}{dt} = 4y(1-y)$



Example 3. $\dfrac{dy}{dt} = e^t \sin y$



Autonomous Differential Equations

A first-order differential equation with independent variable t and dependent variable y is **autonomous** if

$$\frac{dy}{dt} = f(y).$$

The rate of change of $y(t)$ depends only on the value of y.

Examples of autonomous equations: exponential growth model, radioactive decay, logistic population model

Example. $\dfrac{dv}{dt} = -kv + a \sin bt$

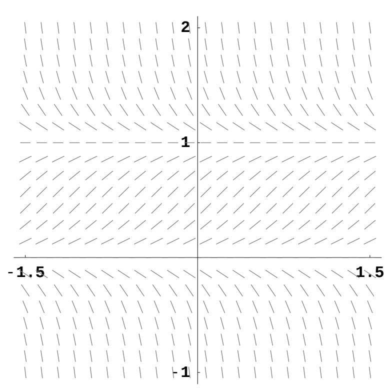
This is a nonautonomous linear differential equation that is related to simple models of voltage in an electric circuit (k, a, and b are parameters).

Comments:

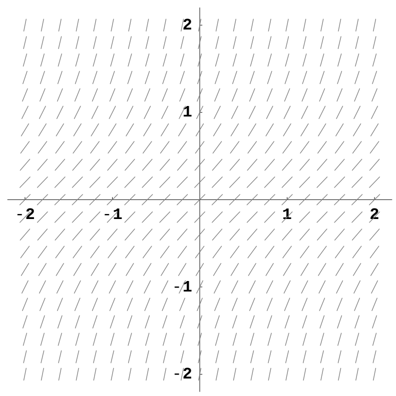
1. Many interesting models in science and engineering are autonomous (but not every model).
2. Every autonomous equation is separable, but the integrals may be impossible to calculate in terms of standard functions.

Basic Fact: Given the graph of one solution to an autonomous equation, we can get the graphs of many other solutions by translating that graph left or right.

Example 1. $\dfrac{dy}{dt} = 4y(1-y)$

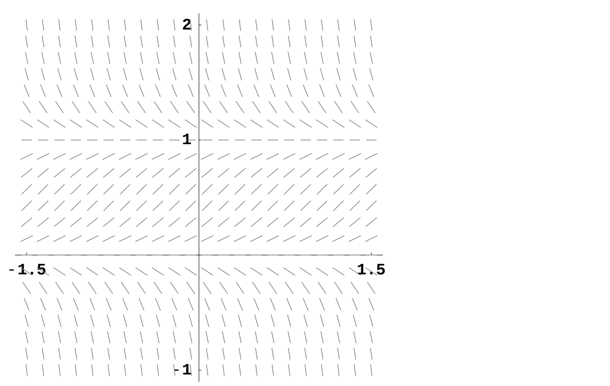


Example 2. $\dfrac{dy}{dt} = 1 + y^2$



The slope field has so much redundant information that we can replace it with the **phase line**. Here's the phase line for our standard example:

Example. $\dfrac{dy}{dt} = 4y(1-y)$



Professor Devaney built a simple Quicktime animation that illustrates how you should interpret this phase line. There is a link to it on our course web page. Also, `PhaseLines` in `DETools` helps you visualize the meaning of the phase line.

Building phase lines

How do we go about building a phase line from a differential equation?

Example $\dfrac{dy}{dt} = y^2 \cos y$