

## Parameters, Qualitative Equivalence, and Bifurcations

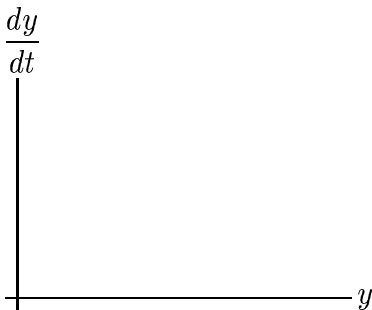
Let's return to the logistic model of population growth

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N}\right)$$

and modify this model to account for constant harvesting:

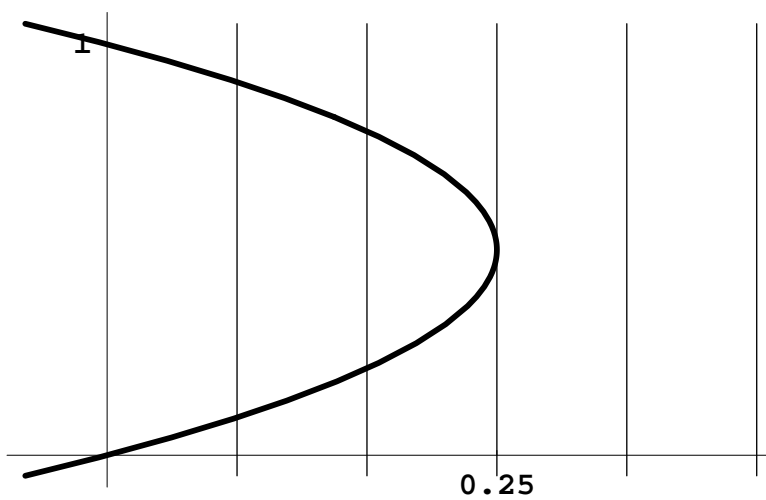
Before we tackle this modification of the logistic model, let's consider an example in which the algebra is simpler.

**Example.**  $\frac{dy}{dt} = y(1 - y) - a$



There is a tool in DETools called `PhaseLines`, and it helps us analyze phase lines and various graphs as we vary certain parameters (the parameter  $a$  in this case).

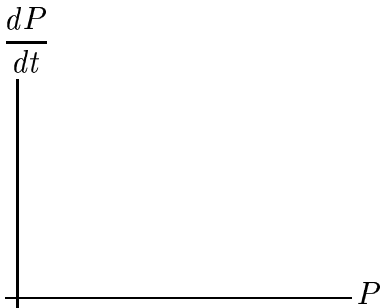
We can summarize the behavior of this one-parameter family of differential equations using a bifurcation diagram.



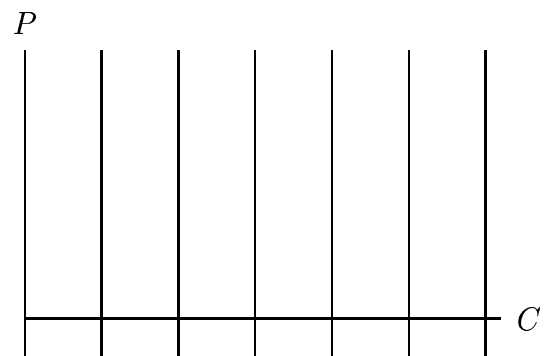
Now let's sketch and interpret the bifurcation diagram for the logistic population model with constant harvesting

$$\frac{dP}{dt} = kP \left(1 - \frac{P}{N}\right) - C.$$

First, let's compute the bifurcation value.



Now we sketch the bifurcation diagram.



What does this diagram say about how we must act if we want fish populations to return to sustainable levels?