Analytic Techniques:

There are few analytic techniques that work for both linear and nonlinear systems.

1. You can always check to see if a given function is a solution (no wrong answers).

   For example, consider the initial-value problem

$$\frac{dx}{dt} = 2y - x \qquad \frac{dy}{dt} = y \qquad (x_0, y_0) = (2, 1).$$

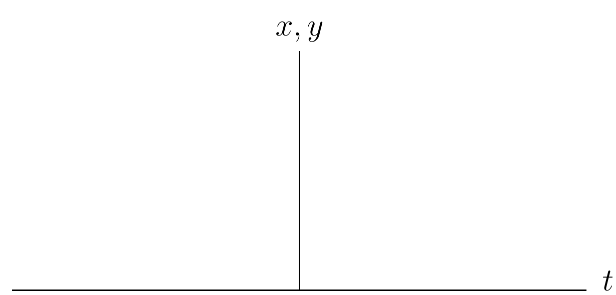
   Using the vector notation

$$\mathbf{Y}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

   we can write this initial-value problem as

$$\frac{d\mathbf{Y}}{dt} = \begin{pmatrix} 2y - x \\ y \end{pmatrix}, \qquad \mathbf{Y}(0) = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

   First, let's see what the solution looks like when we graph it with `HPGSystemSolver`:

Claim: The function

$$\mathbf{Y}(t) = \begin{pmatrix} e^t + e^{-t} \\ e^t \end{pmatrix}$$

solves the initial-value problem.

2. General solution of a partially-decoupled system

**Example.** Consider the previous system

$$\frac{dx}{dt} = 2y - x$$
$$\frac{dy}{dt} = y.$$

We can calculate the general solution using methods we learned for first-order equations:

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Damped Harmonic Oscillator

Let's return to our mass-spring system and add a term that models damping.

Assumption: The damping force is proportional to the speed of the mass and it acts as a restoring force.

The second-order equation

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0$$

and its equivalent system

$$\begin{aligned}\frac{dy}{dt} &= v \\ \frac{dv}{dt} &= -\frac{k}{m}y - \frac{b}{m}v\end{aligned}$$

appear in many applications. On the CD, you will find it in `MassSpring` and `RLCCircuits`, and it has also been used to study biological processes such as the blood glucose regulatory system in humans.

There is a guessing technique for the damped harmonic oscillator

$$m\frac{d^2y}{dt^2} + b\frac{dy}{dt} + ky = 0.$$

**Example.** Consider the harmonic oscillator

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 0.$$

Its characteristic equation is

Let's plot these solutions with `HPGSystemSolver`. What are the corresponding solution curves and component graphs?