Find the arc length of the curve

$$
\mathbf{r}(t)=\cos t \mathbf{i}+\sin t \mathbf{j}+\frac{2}{3} t^{3 / 2} \mathbf{k}
$$

from $t=0$ to $t=4 \pi$.

$$
\begin{aligned}
\vec{r}^{\prime}(t) & =(-\sin t) \vec{\tau}+(\cos t) \vec{j}+\left(t^{1 / 2}\right) \vec{k} \\
\left\|\vec{r}^{\prime}(t)\right\| & =\sqrt{\sin ^{2} t+\cos ^{2} t+t} \\
& =\sqrt{1+t} \\
\text { arc length } & =\int_{0}^{4 \pi} \sqrt{1+t} d t
\end{aligned}
$$

Let $u=1+t \Rightarrow d u=d t$

$$
\begin{aligned}
\int_{0}^{4 \pi} \sqrt{1+t} d t & =\int_{1}^{4 \pi+1} \sqrt{u} d u \\
& =\left[\frac{2}{3} u^{3 / 2}\right]_{1}^{4 \pi+1} \\
& =\frac{2}{3}\left[(4 \pi+1)^{3 / 2}-1\right] \\
& \approx 32.65
\end{aligned}
$$

