Find the arc length of the curve

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \tfrac{2}{3}t^{3/2}\,\mathbf{k}$$

from $t = 0$ to $t = 4\pi$.

$$\vec{r}\,'(t) = (-\sin t)\vec{i} + (\cos t)\vec{j} + (t^{1/2})\vec{k}$$

$$\|\vec{r}\,'(t)\| = \sqrt{\sin^2 t + \cos^2 t + t}$$

$$= \sqrt{1+t}$$

$$\text{arc length} = \int_0^{4\pi} \sqrt{1+t}\; dt$$

Let $u = 1+t \Rightarrow du = dt$

$$\int_0^{4\pi} \sqrt{1+t}\; dt = \int_1^{4\pi+1} \sqrt{u}\; du$$

$$= \left[\frac{2}{3}u^{3/2}\right]_1^{4\pi+1}$$

$$= \frac{2}{3}\left[(4\pi+1)^{3/2} - 1\right]$$

$$\approx 32.65$$