Consider the solid region S under the plane

$$z = \frac{y}{3}$$

and above the rectangle

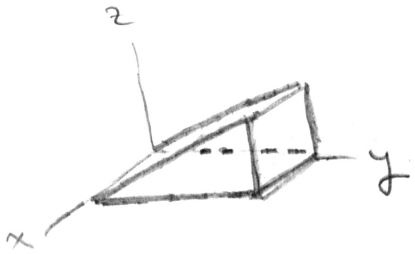
$$R = \{(x, y) \mid 0 \le x \le 2 \text{ and } 0 \le y \le 3\}.$$

Use Cavalieri's principle to compute the volume of S in two different ways:

1. using slices by planes where x is constant, and
2. using slices by planes where y is constant.

1. Each slice with x constant is a triangle of area $\frac{3}{2}$. Since $0 \le x \le 2$, the volume is

$$\left(\frac{3}{2}\right)(2) = 3.$$



2. Each slice with y constant is a rectangle with $0 \le x \le 2$ and $0 \le z \le \frac{y}{3}$. So each rectangle has area $\frac{2}{3}y$.

$$\text{Volume} = \int_0^3 \left(\frac{2}{3}y\right) dy = \left[\frac{1}{3}y^2\right]_0^3$$

$$= 3.$$