Calculate the average value of the function

$$f(x, y) = xe^{xy}$$

over the rectangle

$$R = \{(x, y) \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2\}.$$

$$\text{average value} = \frac{\displaystyle\iint_R xe^{xy}\, dA}{\text{area } R}$$

$$\iint_R xe^{xy}\, dA = \int_0^1 \int_0^2 xe^{xy}\, dy\, dx$$

$$= \int_0^1 \left[e^{xy} \right]_0^2 dx$$

$$= \int_0^1 \left(e^{2x} - 1 \right) dx$$

$$= \left[\tfrac{1}{2} e^{2x} - x \right]_0^1$$

$$= \left(\tfrac{1}{2} e^2 - 1 \right) - \left(\tfrac{1}{2} \right)$$

$$= \tfrac{1}{2} \left(e^2 - 3 \right)$$

$$\text{area } R = 2 \quad \Rightarrow \quad \text{average value} = \frac{e^2 - 3}{4}.$$