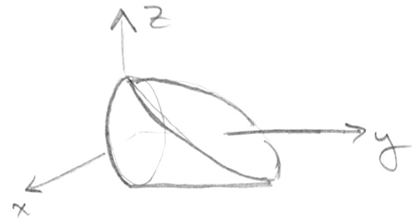
Let Q be the region in $\mathbb{R}^3$ bounded by the cylinder $x^2 + z^2 = 9$, the plane $y + z = 3$, and the plane $y = 0$. Calculate

$$\iiint_Q z\, dV.$$



Given Q, note that it is clear that

$$\iiint_Q z\, dV < 0.$$

Q is y-simple and the projection Q' of Q into the xz-plane is

$$\{(x,z) \mid x^2+z^2 \le 9\}.$$

Therefore

$$\iiint_Q z\, dV = \iint_{Q'} \left(\int_0^{3-z} z\, dy \right) dA$$

$$= \int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} (3z - z^2)\, dz\, dx$$

By geometric considerations, we know that

$$\iint_{Q'} 3z\, dA = 0.$$

So we need only compute

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} -z^2\, dz\, dx = \int_{-3}^{3} -\frac{2}{3}(9-x^2)^{3/2}\, dx =$$

$$-\left[\frac{15}{4} x\sqrt{9-x^2} - \frac{1}{6}x^3\sqrt{9-x^2} + \frac{81}{4}\arcsin\frac{x}{3}\right]_{-3}^{3}$$

$$= -\frac{81}{4}\pi \approx -63.62$$