

Determine the formula $T(x, y)$ for the mapping $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that corresponds to rotation of \mathbb{R}^2 about the point $(2, 1)$ by 90° in the counterclockwise direction.

Desired $T = T_3 \circ T_2 \circ T_1$ where

$T_1 =$ translate plane so that
 $(2, 1) \mapsto (0, 0)$

$T_2 =$ rotate plane about origin
by 90°

$T_3 =$ translate plane so that
 $(0, 0) \mapsto (2, 1)$

We have $T_1(x, y) = (x-2, y-1)$

$$\begin{aligned} T_2(x, y) &= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= (-y, x) \end{aligned}$$

$$T_3(x, y) = (x+2, y+1)$$

So

$$\begin{aligned} T(x, y) &= T_3(1-y, x-2) \\ &= (3-y, x-1). \end{aligned}$$