Use spherical coordinates to compute the volume of the solid that is inside the sphere
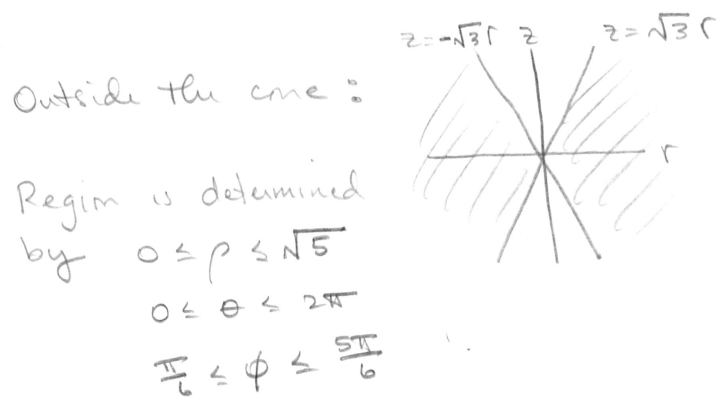
$$x^2 + y^2 + z^2 = 5$$

and "outside" the cone
$$z^2 = 3x^2 + 3y^2.$$

Another way of saying "outside" the cone is

$$z^2 \leq 3x^2 + 3y^2.$$

Outside the cone:

Region is determined
by $0 \leq \rho \leq \sqrt{5}$

$0 \leq \theta \leq 2\pi$

$\frac{\pi}{6} \leq \phi \leq \frac{5\pi}{6}$

Volume $= \iiint\limits_{R} 1 \, dV$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} \int_{\pi/6}^{\frac{5\pi}{6}} \rho^2 \sin\phi \, d\phi \, d\rho \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} \rho^2 \Big[-\cos\phi \Big]_{\pi/6}^{5\pi/6} \, d\rho \, d\theta$$

$$= \int_0^{2\pi} \int_0^{\sqrt{5}} \rho^2 \sqrt{3} \, d\rho \, d\theta$$

$$= \int_0^{2\pi} \sqrt{3} \left[\frac{\rho^3}{3} \right]_0^{\sqrt{5}} d\theta$$

$$= (2\pi)(5)\frac{\sqrt{15}}{3} = \frac{10\pi}{3}\sqrt{15}$$