Compute the center of mass of the region in the first octant enclosed by the sphere

$$x^2 + y^2 + z^2 = 4$$

assuming that the density is constant.

Since the radius is 2 and the density δ is constant, the mass is

$$\frac{\left(\frac{4}{3}\pi\right)(8)(\delta)}{8} = \frac{4}{3}\pi\delta .$$

To calculate the z-coordinate of the center of mass, we must calculate

$$\iiint_R \delta z \, dV = \delta \iiint_R z \, dV$$

Using spherical coordinates, we have

$$\delta \int_0^{\pi/2} \int_0^2 \int_0^{\pi/2} (\rho \cos\phi)(\rho^2 \sin\phi) \, d\phi \, d\rho \, d\theta$$

$$= \delta \int_0^{\pi/2} \int_0^2 \rho^3 \left[\int_0^1 u \, du \right] d\rho \, d\theta$$

$$u = \sin\phi$$

$$= \delta \int_0^{\pi/2} \int_0^2 \frac{\rho^3}{2} \, d\rho \, d\theta = \frac{\delta}{8} \int_0^{\pi/2} \left[\rho^4\right]_0^2 d\theta$$

$$= \left(\frac{\delta}{8}\right)\left(\frac{\pi}{2}\right)(16) = \delta\pi$$

$$\Rightarrow \bar{z} = \frac{\delta\pi}{\frac{4}{3}\pi\delta} = \frac{3}{4} .$$

By symmetry, $\bar{x} = \bar{y} = \bar{z}$.