

Let l_1 be the line through the points $(1, 0, 0)$ and $(1, 2, 2)$, and let l_2 be the line through the points $(1, 1, 1)$ and $(1 + \sqrt{2}, 2, 2)$. Do l_1 and l_2 intersect? If so, at what point do they intersect?

$$\begin{aligned}\vec{D}_1 &= \text{direction vector for } l_1 \\ &= 2\vec{j} + 2\vec{k} \quad \text{or } \vec{j} + \vec{k}\end{aligned}$$

$$\begin{aligned}\vec{D}_2 &= \text{direction vector for } l_2 \\ &= \sqrt{2}\vec{i} + \vec{j} + \vec{k}\end{aligned}$$

$$l_1 : \quad \begin{aligned}x &= 1 \\ y &= t_1 \\ z &= t_1\end{aligned}$$

$$l_2 : \quad \begin{aligned}x &= 1 + \sqrt{2}t_2 \\ y &= 1 + t_2 \\ z &= 1 + t_2\end{aligned}$$

$$l_1 \text{ intersects } l_2 \Leftrightarrow \begin{aligned}t_2 &= 0 \\ t_1 &= 1 + t_2\end{aligned}$$

$\Rightarrow l_1$ and l_2 intersect at $(1, 1, 1)$.