Calculate

$$\int_C (x^2 - y^2)\,dx + (2xy)\,dy$$

where C is the positively-oriented unit (half) circle with $y \geq 0$.

Parameterize C by

$$\vec{r}(t) = (\cos t)\,\vec{i} + (\sin t)\,\vec{j}$$

with $0 \leq t \leq \pi$

Then $dx = (-\sin t)\,dt$

$dy = (\cos t)\,dt$

$$\int_C (x^2 - y^2)\,dx + (2xy)\,dy =$$

$$\int_0^\pi (\cos^2 t - \sin^2 t)(-\sin t) + 2(\cos t)(\sin t)(\cos t)\,dt$$

$$= \int_0^\pi (\cos^2 t)(\sin t) + (\sin^2 t)(\sin t)\,dt$$

$$= \int_0^\pi \sin t\,dt = \left[-\cos t\right]_0^\pi$$

$$= 1 - (-1) = 2.$$