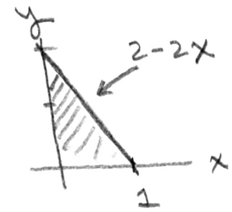
Calculate the coordinates of the center of mass of a homogeneous triangular surface with vertices $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 3)$.

triangle T lies above $T^* =$



on the plane

$$6x + 3y + 2z = 6$$

T is the graph of $z = g(x, y)$ where $g(x, y) = 3 - 3x - \frac{3}{2}y$. So

$\vec{n} = 3\vec{\imath} + \frac{3}{2}\vec{\jmath} + \vec{k}$, and $\|\vec{n}\| = \frac{7}{2}$.

Therefore $dS = \frac{7}{2}\,dA$ where A is area in the xy-plane.

$$\text{area}(T) = \frac{7}{2}\,\text{area}(T^*) = \frac{7}{2}.$$

Then
$$\bar{x} = \frac{\iint_T x\,dS}{\text{area}\,T} = \frac{2}{7}\int_0^1\int_0^{2-2x} x\left(\frac{7}{2}\right)dy\,dx$$
$$= \int_0^1\int_0^{2-2x} x\,dy\,dx$$
$$= \int_0^1 2x - 2x^2\,dx$$
$$= \left[x^2 - \frac{2}{3}x^3\right]_0^1 = \frac{1}{3}$$

Also $\bar{y} = \dfrac{\iint_T y\, dS}{\text{area } T} = \frac{2}{7}\int_0^1 \int_0^{2-2x} y\left(\frac{7}{2}\right) dy\, dx$

$$= \int_0^1 \int_0^{2-2x} y\, dy\, dx$$

$$= \frac{1}{2}\int_0^1 (2-2x)^2\, dy\, dx$$

$$= \frac{1}{2}\int_0^1 4 - 8x + 4x^2\, dx$$

$$= \frac{1}{2}\left[4x - 4x^2 + \frac{4}{3}x^3\right]_0^1$$

$$= \frac{1}{2}\left[4 - 4 + \frac{4}{3}\right] = \frac{2}{3}$$

Also, $6\bar{x} + 3\bar{y} + 2\bar{z} = 6 \Rightarrow$

$$2 + 2 + 2\bar{z} = 6$$

$$2\bar{z} = 2 \Rightarrow \bar{z} = 1.$$