Use spherical coordinates to calculate the flux of the vector field

$$\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$$

across the unit sphere

$$x^2 + y^2 + z^2 = 1.$$

Use the outward pointing orientation.

Parametrize sphere using

$$\Phi(\theta, \phi) = (\sin\phi \cos\theta, \sin\phi \sin\theta, \cos\phi).$$

Then $T_\theta = (-\sin\phi \sin\theta)\vec{\imath} + (\sin\phi \cos\theta)\vec{\jmath}$

$$T_\phi = (\cos\phi \cos\theta)\vec{\imath} + (\cos\phi \sin\theta)\vec{\jmath} + (-\sin\phi)\vec{k}$$

and

$$T_\phi \times T_\theta = \det \begin{bmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \cos\phi\cos\theta & \cos\phi\sin\theta & -\sin\phi \\ -\sin\phi\sin\theta & \sin\phi\cos\theta & 0 \end{bmatrix}$$

$$= (\sin^2\phi \cos\theta)\vec{\imath} + (\sin^2\phi \sin\theta)\vec{\jmath} + (\sin\phi \cos\phi \cos^2\theta + \sin\phi\cos\phi \sin^2\theta)\vec{k}$$

$$= (\sin^2\phi \cos\theta)\vec{\imath} + (\sin^2\phi \sin\theta)\vec{\jmath} + (\sin\phi \cos\phi)\vec{k}.$$

Evaluating the vector field along the sphere, we have

$$\vec{F}(\Phi(\theta, \phi)) =$$

$$(\cos\phi)\vec{\imath} + (\sin\phi)(\sin\theta)\vec{\jmath} + (\sin\phi)(\cos\theta)\vec{k}$$

Flux across $S = \iint_S (\vec{F} \cdot \vec{u})\, dS$ $\quad \vec{u}$ unit normal

$$= \iint_D \vec{F} \cdot \frac{\vec{T}_\phi \times \vec{T}_\theta}{\|\vec{T}_\phi \times \vec{T}_\theta\|} \|\vec{T}_\phi \times \vec{T}_\theta\| \, d\phi \, d\theta$$

where $D = \{(\phi, \theta) \mid 0 \le \phi \le \pi, \ 0 \le \theta \le 2\pi\}$.

$$\text{Flux} = \int_0^{2\pi} \int_0^{\pi} (\sin^2\phi)(\cos\phi)(\cos\theta) + (\sin^3\phi)(\sin^2\theta) + (\sin^2\phi)(\cos\phi)(\cos\theta) \, d\phi \, d\theta$$

$$= 2 \int_0^{2\pi} \int_0^{\pi} (\sin^2\phi)(\cos\phi)(\cos\theta) \, d\phi \, d\theta +$$

$$\int_0^{2\pi} \int_0^{\pi} (\sin^3\phi)(\sin^2\theta) \, d\phi \, d\theta$$

Since $\int_0^{2\pi} (\cos\theta) \, d\theta$, the first integral is 0.

The second integral is equal to

$$\left(\int_0^{2\pi} \sin^2\theta \, d\theta\right)\left(\int_0^{\pi} \sin^3\phi \, d\phi\right) =$$

$$(\pi)\left(\frac{4}{3}\right).$$