

Consider the line integral

$$\int_C -y^3 dx + x^3 dy$$

where C is the positively-oriented unit circle.

1. Parametrize C and convert this line integral into a regular 1-dim integral. Do not evaluate.
2. Use Green's Theorem to convert this line integral into a double integral. Evaluate this integral.

$$\begin{aligned} 1. \quad \vec{r}(t) &= \cos t \vec{i} + \sin t \vec{j} \\ \vec{r}'(t) &= -\sin t \vec{i} + \cos t \vec{j} \end{aligned}$$

$$\int_C -y^3 dx + x^3 dy =$$

$$\int_0^{2\pi} \sin^4 t + \cos^4 t dt$$

$$2. \quad \iint_{0 \leq x^2 + y^2 \leq 1} 3x^2 + 3y^2 dA =$$

$$\int_0^{2\pi} \int_0^1 (3r^2) r dr d\theta =$$

$$\int_0^{2\pi} \int_0^1 3r^3 dr d\theta =$$

$$\frac{3}{4} \int_0^{2\pi} d\theta = \frac{3}{4} (2\pi)$$

$$= \frac{3\pi}{2}.$$