Let C be the curve of intersection of the plane $y + z = 1$ and the cylinder

$$x^2 + y^2 = 1.$$

Orient C using the orientation that is consistent with the normal vector $\mathbf{M} = \mathbf{j} + \mathbf{k}$ for the plane. Use Stokes' Theorem to calculate the line integral

$$\int_C x^2\,dx + xy\,dy + z^3\,dz.$$

We have $\vec{F} = x^2\vec{i} + xy\vec{j} + z^3\vec{k}$.

Then

$$\operatorname{curl}\vec{F} = \nabla\times\vec{F} = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & z^3 \end{bmatrix}$$

$$= y\vec{k}.$$

Let S be the surface determined by the plane and the cylinder.

$$\iint_S \operatorname{curl}\vec{F}\cdot\underbrace{\vec{n}}_{\text{unit normal}}\,dS = \iint_{\underbrace{D}_{\text{projection of } S \text{ into } xy\text{-plane}}} \operatorname{curl}\vec{F}\cdot\vec{m}\,dA$$

$$D = \{(x,y) \mid x^2+y^2 \le 1\}$$

$$\iint_D \operatorname{curl}\vec{F}\cdot\vec{m}\,dA = \iint_D y\,dA \underset{\substack{\text{by symmetry} \\ \text{(or change to polar coordinates)}}}{=} 0$$