$$
\begin{aligned}
& \hline \hline \text { MA } 230 \\
& \hline \hline \text { Consider the vector field } \\
& \qquad \begin{array}{l}
\mathbf{F}(x, y, z)=\left(x^{3}+y \sin z\right) \mathbf{i}+\left(y^{3}+z \sin x\right) \mathbf{j}+(3 z) \mathbf{k} .
\end{array}
\end{aligned}
$$

Use the Divergence Theorem (Gauss' Theorem) to calculate the flux of $\mathbf{F}$ across the surface $S$, where $S$ is the boundary of the solid that is bounded by the hemispheres

$$
z=\sqrt{4-x^{2}-y^{2}}
$$

and

$$
z=\sqrt{1-x^{2}-y^{2}}
$$

and the plane $z=0$.

$$
\begin{aligned}
& \operatorname{div} \vec{F}=3 x^{2}+3 y^{2}+3 \\
& R=\text { regin bid by } \\
& \begin{aligned}
R & =\iiint_{R}(\text { div } \vec{F}) d v \\
& =\iiint_{R} 3 x^{2}+3 y^{2}+3 d v \\
& =3 \iint_{R} x^{2}+y^{2}+1 \text { dV}
\end{aligned}
\end{aligned}
$$

use spherical coordinates to

$$
\begin{aligned}
& \text { Use spherical } \left.\begin{array}{rl}
\text { describe } R=\{(\rho, \phi, \theta) \mid & 1 \leq \rho \leq 2 \\
0 \leqslant 0 \leqslant 2 \pi \\
0 \leqslant \phi \leq \pi / 2
\end{array}\right\} \\
& \begin{aligned}
\text { flux } & =3 \int_{0}^{2 \pi} \int_{0}^{\pi / 2} \int_{1}^{2}\left(\rho^{2} \sin ^{2} \phi+1\right)\left(\rho^{2} \sin \phi\right) d \rho d \phi d \theta \\
& =3 \int_{0}^{2 \pi} \int_{0}^{\pi / 2} \int_{1}^{2}\left(\rho^{4} \sin ^{3} \phi+\rho^{2} \sin \phi\right) d \rho d \phi d \theta \\
& =3 \int_{0}^{2 \pi} \int_{0}^{\pi / 2}\left(\frac{31}{5} \sin ^{3} \phi+\frac{7}{3} \sin \phi\right) d \phi d \theta
\end{aligned}
\end{aligned}
$$

Note that

$$
\begin{aligned}
\sin ^{3} \phi & =\left(\sin ^{2} \phi\right)(\sin \phi) \\
& =\left(1-\cos ^{2} \phi\right)(\sin \phi)
\end{aligned}
$$

and

$$
\begin{aligned}
& \text { and } \quad \int \sin ^{3} \phi d \phi=-\cos \phi+\frac{\cos ^{3} \phi}{3}+C \\
& \begin{aligned}
\text { flax } & =3 \int_{0}^{2 \pi} \frac{31}{5}\left[\cos \phi-\frac{\cos ^{3} \phi}{3}\right]_{\pi / 2}^{0}+\frac{7}{3}[\cos \phi]_{\pi / 2}^{0} d \theta \\
& =3 \int_{0}^{2 \pi}\left(\frac{31}{5}\right)\left(\frac{2}{3}\right)+\left(\frac{7}{3}\right)(1) d \theta \\
& =\int_{0}^{2 \pi}\left(\frac{62}{5}+7\right) d \theta \\
& =(2 \pi)\left(\frac{97}{5}\right)=\frac{194}{5} \pi
\end{aligned}
\end{aligned}
$$

