Let

$$f(x,y) = y\cos x.$$

Find the points on the graph of f where the tangent plane is parallel to the plane

$$x - \sqrt{3}\,y + 2z = -2.$$

$$\frac{\partial f}{\partial x} = -y\sin x \qquad \frac{\partial f}{\partial y} = \cos x$$

Normal vector to the tangent plane is $\vec{T} = (-y\sin x)\vec{i} + (\cos x)\vec{j} - \vec{k}$

Normal vector to plane is

$$\vec{N} = \vec{i} - \sqrt{3}\,\vec{j} + 2\vec{k}$$

The planes are parallel if $\vec{T}$ and $\vec{N}$ are parallel, i.e., $\vec{T} = \lambda\vec{N}$ for some scalar λ. We have

$$\begin{cases} -y\sin x = \lambda \\ \cos x = -\sqrt{3}\,\lambda \\ -1 = 2\lambda \end{cases}$$

$\Rightarrow \lambda = -1/2 \Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow \sin x = \pm\frac{1}{2}$.

We have $x = \pm\frac{\pi}{6} + 2k\pi$. If $x = \frac{\pi}{6} + 2k\pi$, then $y = 1$, and the point is $\left(\frac{\pi}{6} + 2k\pi, 1, \frac{\sqrt{3}}{2}\right)$.

If $x = -\frac{\pi}{6} + 2k\pi$, then $y = -1$ and the point is $\left(-\frac{\pi}{6} + 2k\pi, -1, -\frac{\sqrt{3}}{2}\right)$.