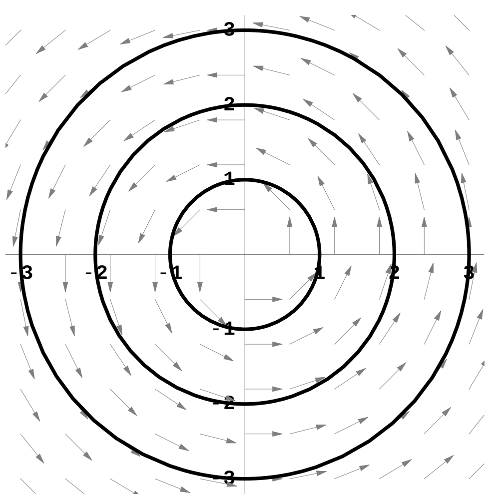
Three examples to illustrate the geometry of complex eigenvalues:

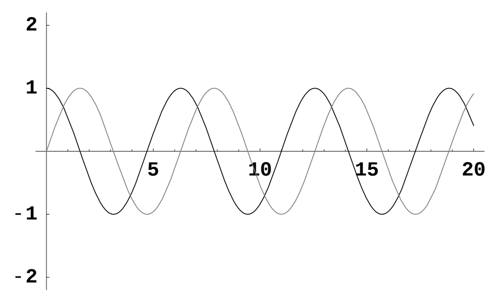
Example 1. $\dfrac{d\mathbf{Y}}{dt} = \mathbf{AY}$ where

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{A}$ is $\lambda^2 + 1$, so the eigenvalues are $\lambda = \pm i$. One eigenvector associated to the eigenvalue $\lambda = i$ is

$$\mathbf{Y}_0 = \begin{pmatrix} i \\ 1 \end{pmatrix}.$$



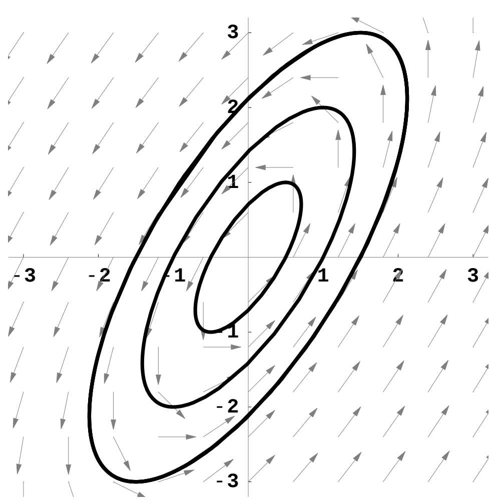


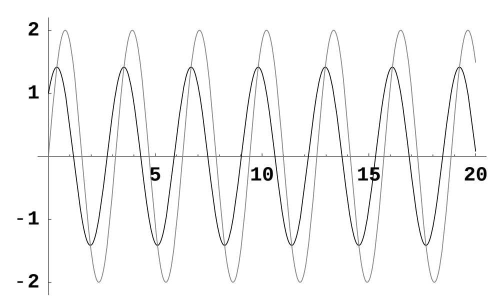
Example 2. $\dfrac{d\mathbf{Y}}{dt} = \mathbf{BY}$ where

$$\mathbf{B} = \begin{pmatrix} 2 & -2 \\ 4 & -2 \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{B}$ is $\lambda^2 + 4$, so the eigenvalues are $\lambda = \pm 2i$. One eigenvector associated to the eigenvalue $\lambda = 2i$ is

$$\mathbf{Y}_0 = \begin{pmatrix} 1+i \\ 2 \end{pmatrix}.$$





Example 3. $\dfrac{d\mathbf{Y}}{dt} = \mathbf{C}\mathbf{Y}$ where

$$\mathbf{C} = \begin{pmatrix} 1.9 & -2 \\ 4 & -2.1 \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{C}$ is $\lambda^2 + 0.2\lambda + 4.01$, so the eigenvalues are $\lambda = -0.1 \pm 2i$. One eigenvector associated to the eigenvalue $\lambda = -0.1 + 2i$ is

$$\mathbf{Y}_0 = \begin{pmatrix} 1 + i \\ 2 \end{pmatrix}.$$