1. (12 points) **Use row operations** to calculate the determinant of the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 2 \\ 3 & -1 & 4 & 6 \\ 2 & -2 & 2 & 4 \\ -2 & 4 & -1 & -1 \end{bmatrix}.$$

2. (16 points) Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & -2 \\ 1 & 1 & 3 & 2 & -1 \end{bmatrix}.$$

Calculate bases for col $\mathbf{A}$ and nul $\mathbf{A}$.

3. (10 points) Let P be the parallelogram in $\mathbb{R}^2$ with vertices $(-1, 0),\ (0, 5),\ (1, -4),\ (2, 1)$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by

$$T(\mathbf{x}) = \begin{bmatrix} 5 & 2 \\ 1 & 1 \end{bmatrix} \mathbf{x}.$$

Calculate the area of $T(P)$.

4. (16 points) **Note that part b of this problem is on the next page.** Let

$$\mathbf{A} = \begin{bmatrix} 0 & -1 & -2 \\ 0 & 1 & 0 \\ 3 & 0 & 6 \end{bmatrix}.$$

(a) Compute $\mathbf{A}^{-1}$. You may use your calculator to double check your answer, but you will not get any credit unless you show enough work so that I can be sure that you can do this problem without your calculator.

4. (continued)

(b) Write $\mathbf{A}^{-1}$ as a product of elementary matrices. You do **NOT** need to multiply the elementary matrices together when you write $\mathbf{A}^{-1}$ as a product.

5. (16 points) **Note that part b of this problem is on the next page.** The trace of a matrix is the sum of its entries along the diagonal. For example, the trace of the 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is $a + d$. Consider the subset S of the vector space $M_{2\times 2}$ of all 2×2 matrices that consists of all matrices whose trace is zero.

   (a) Show that S is a vector subspace of $M_{2\times 2}$.

Problem 5 (continued):

(b) Determine a basis for S. Justify that your answer is a basis.

6. (30 points) Are the following statements true or false? **You will not receive any credit unless you justify your answers.** (Note that there are three more parts to this question on the next two pages.)

   (a) The plane $x_1 + x_2 - 2x_3 = 1$ is a subspace of $\mathbb{R}^3$.

   (b) If the columns of an $n \times n$ matrix $\mathbf{A}$ are linearly independent, then the rows of $\mathbf{A}$ are also linearly independent.

Question 6 (continued):

(c) If $\det(2\mathbf{A}) = 0$ for an $n \times n$ matrix $\mathbf{A}$, then $\mathbf{A}$ is not invertible.

(d) If H and K are subspaces of a vector space V, then their union $H \cup K$ is a subspace of V.

Question 6 (continued):

(e) The transpose of an elementary matrix is elementary.