

Solving systems of linear equations

Consider the linear system of equations

$$2x_1 + x_2 - x_3 = 6$$

$$x_1 + x_2 = 3$$

$$x_1 + x_3 = 1.$$

Let's do a two-variable example more systematically:

$$3x + y = -2$$

$$-x + 3y = 4$$

Elementary row operations on a matrix

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.
2. (Interchange) Interchange two rows.
3. (Scaling) Replace a row by a nonzero multiple of itself.

Two matrices are **row equivalent** if there is a sequence of elementary row operations that transforms one matrix into the other. (Note that row equivalence is an equivalence relation.)

**Theorem.** If the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set.

Now let's return to the original 3-variable example and systematically use row operations:

$$\begin{aligned}2x_1 + x_2 - x_3 &= 6 \\x_1 + x_2 &= 3 \\x_1 + x_3 &= 1.\end{aligned}$$

