More on the diagonalization problem

Last class we learned that the only way that an $n \times n$ matrix $\mathbf{A}$ can be diagonalized is if there is a basis of $\mathbb{R}^n$ of eigenvectors for $\mathbf{A}$.

Let's see why having such as basis is enough to be able to diagonalize $\mathbf{A}$: Suppose $\mathbf{A}$ has n linearly independent eigenvectors

$$\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$$

and let λ_i be the eigenvalue that is associated to $\mathbf{v}_i$. (Note: The λ_i need not be distinct.)

Then we can diagonalize $\mathbf{A}$ using the matrix

$$\mathbf{P} = \left[\begin{array}{c|c|c|c} \mathbf{v}_1 & \mathbf{v}_2 & \quad\ldots\quad & \mathbf{v}_n \end{array}\right].$$

Example. Let

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}.$$

Example. Let

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix}.$$

Now let's return to the unusual matrix that is in the animation.

Example. Consider the matrix

$$\mathbf{B} = \begin{bmatrix} \frac{31}{45} & \frac{19}{45} \\ -\frac{19}{90} & \frac{119}{90} \end{bmatrix}.$$