Fractal examples: Consider the square

$$S = \{(x, y) \,|\, 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$$

and three different ways to "map" S inside of itself.

Solving systems of linear equations

Consider the linear system of equations

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 6 \\ x_1 + x_2 &= 3 \\ x_1 + x_3 &= 1. \end{aligned}$$

Let's do a two-variable example more systematically:

$$\begin{aligned} 3x + y &= -2 \\ -x + 3y &= 4 \end{aligned}$$

Elementary row operations on a matrix

1. (Replacement) Replace one row by the sum of itself and a multiple of another row.
2. (Interchange) Interchange two rows.
3. (Scaling) Replace a row by a nonzero multiple of itself.

Two matrices are **row equivalent** if there is a sequence of elementary row operations that transforms one matrix into the other. (Note that row equivalence is an equivalence relation.)

Theorem. If the augmented matrices of two linear systems are row equivalent, then the two systems have the same solution set.

Now let's return to the original 3-variable example and systematically use row operations:

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 6 \\ x_1 + x_2 &= 3 \\ x_1 + x_3 &= 1. \end{aligned}$$