Today we discuss

1. the topics covered in MA 242 in general terms,
2. how this course will operate,
3. two examples of applications of linear algebra, and
4. we get started on the problem that reappears repeatedly during the first three weeks of the semester—solving systems of linear equations.

Rough Outline of MA 242

1. Linear Equations and Transformations
   (a) row reduction
   (b) solution sets of linear equations
   (c) linear transformations
2. Matrix Algebra
   (a) matrix operations
   (b) invertible matrices
3. Determinants
   (a) definition and properties
   (b) geometric interpretation
4. Abstract vector spaces
   (a) vector spaces and subspaces
   (b) bases and dimension
5. Eigenvalues and eigenvectors
   (a) eigenspaces
   (b) diagonalization
6. Orthogonal sets and matrices

Linear programming example:

Vitamin	Food 1	Food 2	Required Amount
A	30 units/ounce	20 units/ounce	120 units
B	40 units/ounce	10 units/ounce	80 units
C	20 units/ounce	40 units/ounce	100 units
Cost	10 cents/ounce	15 cents/ounce	

Fact. The cost function $c(x, y)$ is minimized at a vertex of the boundary of the feasible set.

Another application of linear algebra: Generating fractals via iterated affine transformations

Consider the square $S = \{(x, y) \mid 0 \leq x \leq 1,\ 0 \leq y \leq 1\}$ and three different ways to "map" S inside of itself.

To iterate these three affine transformations, we pick a point (x_0, y_0) in the square S at random and a transformation T_1 from among the three transformations A_1, A_2, or A_3 at random. Then we compute the point

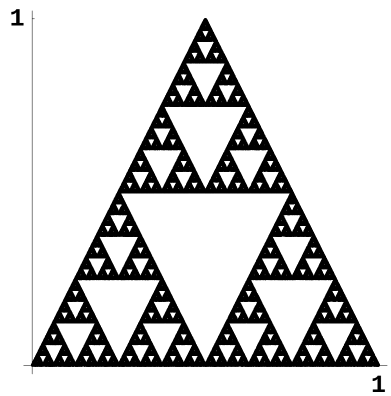
$$(x_1, y_1) = T_1(x_0, y_0).$$

Next we pick a second transformation T_2 from among A_1, A_2, or A_3 at random and we compute

$$(x_2, y_2) = T_2(x_1, y_1).$$

Finally we "iterate" this process. At the kth step, we pick a transformation T_k from A_1, A_2, or A_3 at random, and we compute the point

$$(x_k, y_k) = T_k(x_{k-1}, y_{k-1}).$$

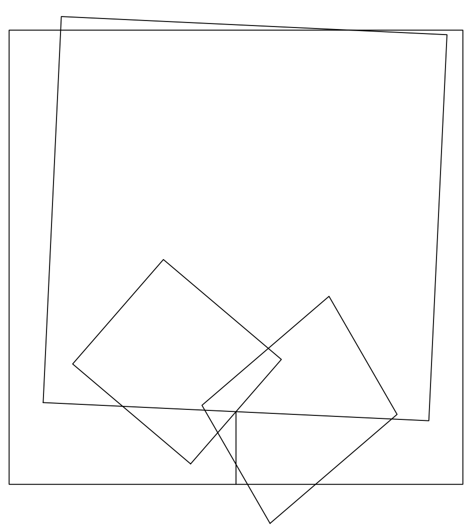


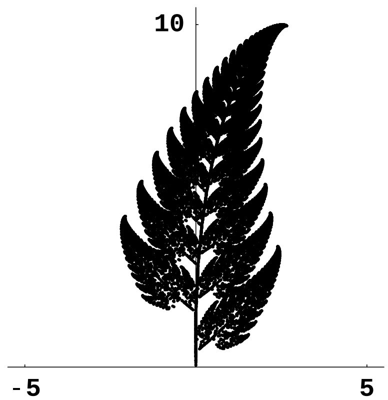
Barnsley's fern

This fractal by Michael Barnsley is produced the same way using the following four transformations:

$$
\begin{aligned}
A_1(x, y) &= (0, 0.16y) \\
A_2(x, y) &= (0.2x - 0.26y, 0.23x + 0.22y + 1.6) \\
A_3(x, y) &= (-0.15x + 0.28y, 0.26x + 0.24y + 0.44) \\
A_4(x, y) &= (0.85x + 0.04y, -0.04x + 0.85y + 1.6)
\end{aligned}
$$

Transformation A_1 is used 1% of the time. Transformations A_2 and A_3 are each used 7% of the time, and transformation A_4 is used 85% of the time.





Solving systems of linear equations

Consider the system of linear equations

$$\begin{aligned} 2x_1 + x_2 - x_3 &= 6 \\ x_1 + x_2 &= 3 \\ x_1 + x_3 &= 1. \end{aligned}$$

How do we interpret this result geometrically?