

MA 713
Reading and Exercises
Week ending April 20

Reading:

Class 36 (4/18): Ahlfors pp. 193–197

Class 37 (4/20): Ahlfors pp. 219–223

Exercises to be submitted for grading on Friday, April 27:

Leftover exercises

Ahlfors Exercises 3 (d)–(f) on p. 161

Ahlfors Exercise 1 on p. 166 (hard)

Class 36 (4/18):

Additional Exercise 10: Show that

$$\frac{\pi}{2} = \left(\frac{2}{1}\right) \left(\frac{2}{3}\right) \left(\frac{4}{3}\right) \left(\frac{4}{5}\right) \left(\frac{6}{5}\right) \left(\frac{6}{7}\right) \left(\frac{8}{7}\right) \cdots$$

Additional Exercise 11:

a. Let $0 < |a| < 1$ and $|z| \leq r < 1$. Show that

$$\left| \frac{a + |a|z}{(1 - \bar{a}z)a} \right| \leq \frac{1 + r}{1 - r}$$

b. Let $\{a_n\}$ be a sequence of numbers such that $0 < |a_n| < 1$ and $\sum(1 - |a_n|) < \infty$. Show that the infinite product

$$\prod_{n=1}^{\infty} \frac{|a_n|}{a_n} \left(\frac{a_n - z}{1 - \bar{a}_n z} \right)$$

converges to an analytic function that maps the open unit disk \mathbb{D} to itself.