MA 713
Reading and Exercises
Week ending February 23

Reading:

Class 15 (2/20): Ahlfors pp. 120–123

Class 17 (2/23): Ahlfors pp. 124–126

Exercises to be submitted for grading on Friday, March 2:

Class 15 (2/20):

Additional Exercise 5: Let g be a continuous complex-valued function defined on the unit circle $|z| = 1$. Define

$$f(z) = \begin{cases} g(z) & \text{for } |z| = 1; \\ \dfrac{1}{2\pi i} \displaystyle\int_\gamma \frac{g(w)}{w - z}\, dw & \text{for } |z| < 1, \end{cases}$$

where γ is the positively-oriented unit circle. Is $f(z)$ continuous on the closed unit disk $|z| \leq 1$?

Class 16 (2/21):
Ahlfors Exercises 2 and 4 on p. 123

Additional Exercise 6: Suppose that $f(z)$ is entire and that the harmonic function $u(x, y) = \text{Re}[f(z)]$ is bounded above on $\mathbb{R}^2$, i.e., there exist an M such that $u(x, y) \leq M$ for all $(x, y) \in \mathbb{R}^2$. Show that $u(x, y)$ is constant on $\mathbb{R}^2$.

Class 17 (2/23):

Additional Exercise 7: In the second paragraph on p. 126, Ahlfors derives expression (29)

$$f_n(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)\, d\zeta}{(\zeta - a)^n (\zeta - z)}$$

for the function $f_n(z)$ in the finite Taylor expansion of $f(z)$. That derivation is very succinct. Write a similar derivation in which you verify all of the steps that Ahlfors leaves to the reader.