MA 412 August 3, 1988

Computing Residues

Let

$$f(z) = \frac{g(z)}{h(z)}$$

and suppose that z_0 is a zero of $g(z)$ of order m and z_0 is a zero of order n of $h(z)$ (m and n are nonnegative integers). The following is a quide to the computation of the residue b of $f(z)$ at z_0. Note that essential singularities do not arise in this manner.

1. If $m \geq n$, then z_0 is a removable singularity and $b = 0$.
2. Simple poles, i.e., $n = m + 1$:

$$b = (m+1)\,\frac{g^{(m)}(z_0)}{h^{(m+1)}(z_0)}$$

3. Double poles when $m = 0$ (and therefore $n = 2$):

$$b = 2\,\frac{g'(z_0)}{h''(z_0)} - \frac{2}{3}\,\frac{g(z_0)h'''(z_0)}{[h''(z_0)]^2}$$

4. Pole of order n (i.e. $g(z_0) \neq 0$) and $h(z) = (z - z_0)^n$:

$$b = \frac{g^{(n-1)}(z_0)}{(n-1)!}$$