MA 771 Exercises

1.1. Let R_θ be rotation of the circle S^1 by $2\pi\theta$ radians. In other words, if S^1 is thought of as all complex numbers z such that $|z| = 1$, then $R_\theta(z) = e^{2\pi i\theta}z$. Describe the suspension M of $R_\theta : S^1 \to S^1$ and the dynamics of the suspended flow $\phi_t : M \to M$.

1.2. Let $X = [0, 1]$. Define $f : X \to X$ by $f(x) = 1 - x$. Describe the suspension M of $f : X \to X$ and the dynamics of the suspended flow $\phi_t : M \to M$.

1.3. Let f be complex conjugation of the circle S^1. In other words, if S^1 is thought of as all complex numbers z such that $|z| = 1$, then $f(z) = \overline{z}$. Describe the suspension M of $f : S^1 \to S^1$ and the dynamics of the suspended flow $\phi_t : M \to M$.

1.4. Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map

$$L\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & 1 \\ 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Show (using estimates) that

$$L^n\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

as $n \to \infty$.

1.5. Give an example of $f : X \to X$ where

$$\bigcup_{p \in X} \omega(p) \neq \Omega.$$

1.6. Give an example of a compact space X and a map $f : X \to X$ such that

$$\Omega(f|\Omega) \neq \Omega(f).$$

1.7. Suppose that X is a compact space and $f : X \to X$ is a homeomorphism. If U is a neighborhood of $\Omega(f)$ and $x \in X$, show that there exists an integer N such that $f^n(x) \in U$ for all $n \geq N$.

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