Answers to Even-Numbered Homework Problems, Section 1.1

4. Row reducing the augmented matrix corresponding to the linear system \( x_1 - 5x_2 = 1, \) \( 3x_1 - 7x_2 = 5 \), one finds

\[
\begin{bmatrix}
1 & -5 & 1 \\ 3 & -7 & 5
\end{bmatrix} \sim \ldots \sim 
\begin{bmatrix}
1 & 0 & \frac{9}{4} \\ 0 & 1 & \frac{7}{4}
\end{bmatrix}.
\]

Hence, the point of intersection is given by \((x_1, x_2) = (\frac{9}{4}, \frac{1}{4})\).

14. The augmented matrix corresponding to the given linear system is

\[
\begin{bmatrix}
1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0
\end{bmatrix},
\]

which can be transformed to the reduced echelon form

\[
\begin{bmatrix}
1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1
\end{bmatrix}.
\]

Therefore, the solution is \((x_1, x_2, x_3) = (2, -1, 1)\).

20. Since

\[
\begin{bmatrix}
1 & h & -3 \\ -2 & 4 & 6
\end{bmatrix} \sim 
\begin{bmatrix}
1 & h & -3 \\ 0 & 4 + 2h & 0
\end{bmatrix}
\]

and since the second equation \((4+2h)x_2 = 0\) has a solution for every value of \(h\), the system is consistent for all \(h\).

28. Row reduce the augmented matrix of the given system, which shows

\[
\begin{bmatrix}
a & b & f \\ c & d & g
\end{bmatrix} \sim \ldots \sim 
\begin{bmatrix}
1 & \frac{b}{a} & \frac{f}{a} \\ 0 & d - \frac{c}{a} & g - \frac{c}{a}
\end{bmatrix}.
\]

(Here, one can divide by \(a\), since we assumed \(a \neq 0\).) Since the system is consistent for all possible \(f, g\) by assumption and since \(g - \frac{c}{a}\) can be nonzero, \(d - \frac{c}{a}\) must be nonzero. Therefore, \(ad - bc \neq 0\) must hold.

34. Row reducing the system of equations from 33., we find

\[
\begin{bmatrix}
4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40
\end{bmatrix} \sim 
\begin{bmatrix}
-1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30
\end{bmatrix} \sim \ldots \sim 
\begin{bmatrix}
1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 27.5 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22.5
\end{bmatrix}.
\]

Hence, the solution is \((T_1, T_2, T_3, T_4) = (20, 27.5, 30, 22.5)\).