Answers to Even-Numbered Homework Problems, Section 1.9

2.

\[ A = [T(e_1) \ T(e_2) \ T(e_3)] = \begin{bmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{bmatrix}. \]

6. \( T(e_1) = e_1, \ T(e_2) = e_2 + 3e_1 = (3, 1), \) and

\[ A = [T(e_1) \ T(e_2)] = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}. \]

20. Since \( T(\mathbf{x}) \) has 1 entry, \( A \) has 1 row. Since \( \mathbf{x} \) has 4 entries, \( A \) has 4 columns. Hence,

\[ T(\mathbf{x}) = (2x_1 + 3x_2 - 4x_4) = [2 \ 0 \ 3 \ -4] \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \]

and \( A = [2 \ 0 \ 3 \ -4]. \)

22. Since

\[ T(\mathbf{x}) = \begin{pmatrix} x_1 - 2x_2 \\ -x_1 + 3x_2 \\ 3x_1 - 2x_2 \end{pmatrix} = x_1 \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}, \]

\[ = \begin{bmatrix} 1 & -2 \\ -1 & 3 \\ 3 & -2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \]

\( T(x_1, x_2) = (-1, 4, 9) \) is solved by row reducing the augmented matrix

\[ \begin{bmatrix} 1 & -2 & -1 \\ -1 & 3 & 4 \\ 3 & -2 & 9 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}. \]

Therefore, \((x_1, x_2) = (5, 3).\)

26. The standard matrix \( A \) of \( T \) in Exercise 2 is \( 2 \times 3 \). Its columns are linearly dependent, since \( A \) has more columns than rows. So \( T \) is not one-to-one, by Theorem 12. Also, \( A \) is row equivalent to

\[ \begin{bmatrix} 1 & 4 & -5 \\ 0 & -19 & 19 \end{bmatrix}, \]

which shows that the rows of \( A \) span \( \mathbb{R}^2 \). (There is a pivot position in every row.) Hence, \( T \) is onto.

34. The transformation \( T \) maps \( \mathbb{R}^n \) onto \( \mathbb{R}^m \) if and only if for each \( \mathbf{y} \) in \( \mathbb{R}^m \) there exists an \( \mathbf{x} \) in \( \mathbb{R}^n \) such that \( T(\mathbf{x}) = \mathbf{y}. \)