24. Since \( x \) is an eigenvector of \( A \),
\[
    x^T A x = x^T (\lambda x) = \lambda x^T x.
\]
Since \( z \) is real and non-negative for every complex number \( z \in \mathbb{C} \), \( x^T x \) is real. Now, since \( x^T x \) is real by Exercise 23, so is \( \lambda \).

Next, write \( x = u + iv \) for some real vectors \( u \) and \( v \), and compute
\[
    A x = A(u + iv) = A u + iA v
\]
and
\[
    \lambda x = \lambda u + i\lambda v.
\]
The real part of \( A x \) is \( A u \), because the entries in \( A \), \( u \), and \( v \) are all real. The real part of \( \lambda x \) is \( \lambda u \), because \( \lambda \) and the entries in \( u \) and \( v \) are all real. Since \( A x \) and \( \lambda x \) are equal, their real parts are equal, too. Thus, \( A u = \lambda u \), which shows that the real part of \( x \) is an eigenvector of \( A \).

26. (a) If \( \lambda = a - bi \), then
\[
    A v = \lambda v
    = (a - bi)(\text{Re} v + i\text{Im} v)
    = (a\text{Re} v + b\text{Im} v) + i(a\text{Im} v - b\text{Re} v).
\]
By Exercise 25,
\[
    A(\text{Re} v) = \text{Re} A v = a\text{Re} v + b\text{Im} v,
    A(\text{Im} v) = \text{Im} A v = -b\text{Re} v + a\text{Im} v.
\]

(b) Let \( P = [\text{Re} v \text{ Im} v] \). By (a),
\[
    A(\text{Re} v) = P \begin{pmatrix} a \\ b \end{pmatrix} \quad \text{and} \quad A(\text{Im} v) = P \begin{pmatrix} -b \\ a \end{pmatrix}.
\]
Hence,
\[
    AP = [A(\text{Re} v) \ A(\text{Im} v)]
    = \left[ P \begin{pmatrix} a \\ b \end{pmatrix} \ P \begin{pmatrix} -b \\ a \end{pmatrix} \right]
    = P \begin{pmatrix} a & -b \\ b & a \end{pmatrix}
    = PC.
\]