20. Let

\[ \mathbf{u} = \begin{pmatrix} -\frac{2}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \\ 0 \end{pmatrix}. \]

Since \( \mathbf{u} \cdot \mathbf{v} = 0 \), \( \{\mathbf{u}, \mathbf{v}\} \) is an orthogonal set. However, \( \|\mathbf{u}\|^2 = \mathbf{u} \cdot \mathbf{u} = 1 \) and \( \|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v} = \frac{5}{9} \neq 1 \), so \( \{\mathbf{u}, \mathbf{v}\} \) is not an orthonormal set. The vector \( \mathbf{v} \) can be normalized, with

\[ \tilde{\mathbf{v}} = \frac{1}{\|\mathbf{v}\|} \mathbf{v} = \frac{3}{\sqrt{5}} \mathbf{v} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \\ 0 \end{pmatrix}, \]

so that \( \{\mathbf{u}, \tilde{\mathbf{v}}\} \) is an orthonormal set. (Note that \( \mathbf{u} \) is already a unit vector.)

26. A set of \( n \) nonzero orthogonal vectors must be linearly independent by Theorem 4, so if such a set spans \( W \), it is a basis for \( W \). Since \( W \) is therefore an \( n \)-dimensional subspace of \( \mathbb{R}^n \), it must be equal to \( \mathbb{R}^n \) itself.

28. If \( U \) is an \( n \times n \) orthogonal matrix, then \( I_n = U U^{-1} = U U^T \). Since \( U \) is the transpose of \( U^T \), that is, since \( (U^T)^T = U \), Theorem 6 applied to \( U^T \) says that \( U^T \) has orthogonal columns. In particular, the columns of \( U^T \) are linearly independent and hence form a basis for \( \mathbb{R}^n \), by the Invertible Matrix Theorem. That is, the rows of \( U \) form an orthonormal basis for \( \mathbb{R}^n \).