Solutions to Assignment II

1.a

$$\begin{bmatrix} 0 \\ \frac{5}{2} \\ -\frac{1}{2} \end{bmatrix}$$

1.b

$$5x_1 + 4x_2 + 3x_3 + 2x_4 = 1$$

$$9x_1 + -2x_2 + 4x_3 + 6x_4 = 0$$

$$11x_2 + 5x_3 + x_4 = 7$$

$$9x_1 + 6x_2 + 2x_3 + 4x_4 = -1$$

2.a $1.v_1 + 0.v_2 + 0.v_3 + 1.v_4 = 0$
This implies that the equation $c_1.v_1 + c_2.v_2 + c_3.v_3 + c_4.v_4 = 0$ has solutions $[c_1, c_2, c_3, c_4]$=$[1, 0, 0, 1]$ which is nontrivial. Hence the set $\{v_1, v_2, v_3, v_4\}$ is linearly dependent.

2.b One more vector. It can be anyvector which is not in Span$\{v_1, v_2\}$.
For example $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, *etc.*

2.c

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

3.a

$$T(u + u + u + u) = T(u) + T(u) + T(u) + T(u)$$

3.b

$$\begin{bmatrix} 15 \\ -9 \\ -12 \end{bmatrix}.$$

3.c

$$T(\mathbf{0}) = \mathbf{0} \text{ fails.}$$

4.a

$$det 2A = 2^n .5$$

4.b After elementary row operations $R_4' = R_4 - R_3$ and $R_4' = R_4 - R_3$ the fourth row will become a row of all 0's. Hence the determinant is zero.

Assignment IV

MA 242

Due on 24^{th} July

- Prove that for any $3X3$ **Triangular** matrix A the product of the eigen values (if it exists) is same as the value of the $detA$.
  Then show that this result holds for nXn triangular matrix.
  (Note:- Do not use any concept beyond chapter 5.1)

- Suppose the solutions of a homogeneous system of **five** linear equations in **six** unknowns(variables) are all combinations of two non zero coloumns. Will the system necessarily have a solution for every possible choice of constants on the right hand side of the equation?

- Show that any linearly independent set of **n** vectors in R^n is a basis for R^n.

- Write the co-ordinate vectors first and then with the help of these co-ordinate vectors test the linear independence of the set of polynomials

$$(1-t)^3, (2-3t)^2, 3t^2-4t^3.$$