Test 2

Things to Remember

- A **Vectorspace** is a nonempty set **V** of objects, called vectors, on which are defined two operations, called addition and multiplication by scalars(real numbers),subject to the tex axiomslisted below. The axioms must hold for all vectors **v**, **u**, **w** in **V** and for all scalars c and d.
  1. The sum $\mathbf{v} + \mathbf{u} \in \mathbf{V}$.
  2. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$.
  3. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$.
  4. There is a **zero** vector **0** in V such that $\mathbf{u} + \mathbf{0} = \mathbf{u}$.
  5. For each **u** in V, there is a vector **-u** in V such that $\mathbf{u} + (\text{-}\mathbf{u}) = \mathbf{0}$.
  6. The scalar multiple of **u** bu c, denoted by $c\mathbf{u}$, is in V.
  7. c$(\mathbf{u} + \mathbf{v}) = (c\mathbf{u} + c\mathbf{v})$.
  8. $(c + d)(\mathbf{u}) = (c\mathbf{u} + d\mathbf{u})$.
  9. $c(d\mathbf{u}) = (cd)\mathbf{u}$.
  10. $1\mathbf{u} = \mathbf{u}$.

- A **Subspace** of a vector space V is a subset H of V that has three properties.
  1. The zero vector of V is in H.
  2. H is closed under vector addition. That is, for each **u** and **v** in H , the sum $\mathbf{u} + \mathbf{v}$ is in H.
  3. H is closed under scalar multiplication. That is, for each **u** in H and for each scalar c , the vector c**u** is in H.

- **Span**$\{v_1, v_2, ..., v_p\}$ is the set of vectors that can be written as linear combinations of $\{v_1, v_2, ..., v_p\}$.

- If $v_1, v_2, ..., v_p$ are in a vector space V, the Span$\{v_1, v_2, ..., v_p\}$ is a subspace of V.

- The **Null Space** of an mxn matrix A, denoted by $NulA$,is the set of all solutions to the homogeneous equation A**x**=**0**. In set notation

$$NulA = \{\mathbf{x}\text{: } \mathbf{x} \text{ is in } R^n \text{ and A}\mathbf{x}\text{=0}\}.$$

- The null space of an mxn matrix A is a subspace of R^n.
- The **Coloumn Space** of an mxn matrix A, denoted by $ColA$, is the set of all linear combinations of the coloumns of A. If A=[a_1,a_2,...,a_n], then

$$ColA = \text{Span}\{a_1,a_2,...,a_n\}$$

- The coloumn space of an mxn matrix A is a subspace of R^m.
- The coloumn space of mxn matrix A is all of R^m if and only if the equation $A\mathbf{x} = b$ has a solution for each **b** in R^n.
- Let T be a linear transformation from the vector space V into the vector space W.The NulT is known as **KernalT**.

$$\text{KernalT} = \{\mathbf{x}\text{: } \mathbf{x} \text{ is in V and T}\mathbf{x}\text{=0}\}.$$

- KernalT is a subspace of Vector Space V.
- An indexed set of vectors $\{v_1,...,v_p\}$ in v is said to be linearly independent if the vector equation c_1v_1+ c_2v_2+...+ c_pv_p= **0**. has only the trivial solution ,c_i's are zero. Otherwise Linearly dependent.
- **Basis**:- Let H be the subspace of a vector space V. An indexed set of vectors $\mathcal{B} = \{b_1, b_2, b_3, ..., b_p\}$ in V is a **basis** for H if
  - $\mathcal{B}$ is linearly independent set.
  - $H = Span\mathcal{B}$.
- $\{1, t, t^2, ..., t^n\}$ spans P_n.
- Let $S = \{v_1, v_2, ..., v_p\}$ be a set in V and $H = Span\{v_1, v_2, ..., v_p\}$,then
  - If v_k is the linear combination of the other vectors in S, then the set $S - \{v_k\}$ still spans H.
  - If $H \neq \{0\}$, then some subset of S is a basis for H.

- Elementary row operations on a matrix do not affect the linear dependence relation among the coloumns of the matrix.
- The pivot coloumns of a matrix A form a basis of coloumn for $ColA$.
- $\mathcal{B} = \{b_1, b_2, b_3, ..., b_p\}$ be a basis for a vector space V.Then for each **x** in V, there exists a unique set of scalars c_1,...,c_n such that

$$\mathbf{x}=c_1b_1+c_2b_2+...+c_nb_n$$

- **Co-ordinates** Suppose the set $\mathcal{B} = \{b_1, b_2, b_3, ..., b_n\}$is a basis for V and **x** is in V. The co-ordinates of **x** relative to the basis $\mathcal{B}$ is the $nx1$ matrix $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \\ . \\ . \\ . \\ c_n \end{bmatrix}$, such that $\mathbf{x}=c_1b_1+c_2b_2+...+c_nb_n$.
- $[\mathbf{P}]_{\mathcal{B}} = [b_1b_2...b_n]$ is known as the **change of co-ordinate matrix**.
- $\mathcal{B} = \{b_1, b_2, b_3, ..., b_p\}$ in V is a **basis** for a vector space V. Then the co-ordinate mapping $\mathbf{x} \rightarrow [\mathbf{x}]_{\mathcal{B}}$ is a one - one transformation from V onto R^n.
- If a vector space V has a basis $\mathcal{B} = \{b_1, b_2, b_3, ..., b_n\}$, then any set in V containing more than n vectors must be linearly independent.
- P^n is isomorphic to R^{n+1}.
- If a vector space has a basis of n vectors, then every basis of V must consist of exactly n vectors.
- **DimV** = Number of vectors in a basis for V.
- If **DimV** $< \infty$ then V is said to be finite Dimensional.
- Let H be the subspace of a finite dimensional vector space V. Any linearly independent set in H can be expanded, if necessary, to a basis of H . Also H is finite dimensional and

$$DimH \leq DimV$$

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The Basis Theorem Let V be the p-dimensional vector space , $p \geq 1$.Any linearly independent set of exactly p elements in V is automatically a basis for V. Any set of exactly p elements that spans V is automatically a basis for V.

- The dimension of $NulA$ is the number of free variables in the equation $A\mathbf{x} = 0$, and the dimension of coloumn A is the number of pivot coloumns in A.
- **Rank** A is the dimension of the **coloumn space** of A.
- **Rank Theorem** $\mathbf{RankA} + \mathbf{DimNulA} = n$,where A is a mxn matrix.
- **The Invertible Matrix Theorem**
  Let A be nxn matrix. Then the following statements are equivalent to the statement that A is invertible matrix.
  - A is invertible.
  - The coloumns of A forms a basis of R^n.
  - $ColA = R^n$.
  - Dim Col$A$=n
  - Rank A = n.
  - $NulA = \mathbf{0}$
  - Dim $NulA$=0
- **Eigenvectors and Eigenvalues**:-

  An Eigenvector of an nxn matrix A is a nonzero vector $\mathbf{x}$ such that $A\mathbf{x} = \lambda\mathbf{x}$ for some scalar λ.
  A scalar λ is called an **eigenvalue** of A if there is a nontrivial solution $\mathbf{x}$ of $A\mathbf{x} = \lambda\mathbf{x}$; such an $\mathbf{x}$ is called an Eigenvector corresponding to λ.
- The **eigenvalues** of a **triangular matrix** are the entries on its main diagonal.
- Let A be an nxn matrix. Then A is invertible if and only if **0** is not an eigenvalue of A.

- If x_1 and x_2 are the roots of a quadratic polynomial then the polynomial can be represented as $(t - x_1)(t - x_2) = t^2 - (x_1 + x_2)t + x_1 x_2$. Its is a polynomial in P_2.