Things to Remember for Test III

1. An **eigenvector** of an nXn matrix A is nonzero vector such that $Ax = \lambda x$ for some scalar λ. This λ is called an eigenvalue of A if there is a nontrivial solution $\mathbf{x}$ of $Ax = \lambda x$; such an $\mathbf{x}$ is called an eigenvector corresponding to λ.

2. The solution set of the equation (A-λI)$\mathbf{x} = 0$ forms the Eigen Space of A corresponding to λ.

3. If v_1,...,v_n are eigenvectors that correspond to distinct eigen values λ_1 ,..., λ_n of an nXn matrix A , then the set $\{v_1, ..., v_r\}$ is linearly independent.

4. Det(A-λI) $= 0$ is known as the characteristic equation of an nXn matrix A with λ as eigenvalue.

5. **Similar Matrices**:- If A and B are nXn matrices , then A is similar to B if there is an invertible Matrix P such that $P^{-1}AP = B$.

6. A square Matrix A is said to be Diagonalizable if A is similar to a Diagonal Matrix.

7. An nXn matrix A is diagonalizable if and only if A has n linearly independent eigenvectors. This is also known as eigenvector basis.

8. An nXn matrix with n distinct eigenvalues is diagonalizable.

9. $\begin{bmatrix} u_1 & u_2 & u_3 & . & . & .u_n \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ . \\ . \\ . \\ v_n \end{bmatrix} = u_1v_1 + u_2v_2 + u_3v_3 + ... + u_nv_n$. This is denoted as $u.v$ and is known as **dot product or inner product** between two vectors in R^n.

10. Let u , v and w be vectors in R^n, and let c be a scalar. Then
    (a) $u.v = v.u$
    (b) $(u + v).w = u.w + v.w$
    (c) $(cu).v = c(u.v) = u.(cv)$
    (d) $u.u \geq 0$, and $u.u = 0$ if and only if $u = 0$.

11. **Norm** is the length of the vector **v** and is wrriten as

$$\|v\| = \sqrt{v.v}.$$

12. For vectors v and u in R^n the distance between u and v is written as

$$dist(u, v) = \|u - v\|$$

13. Two vectors u and v in R^n are **orthogonal** to each other if $u.v = 0$.

14. **Pythagorean Theorem** Two vectors u and v are orthogonal if and only if

$$\|u + v\|^2 = \|u\|^2 + \|v\|^2.$$

15. The orthogonal complement of W is denoted by $W^\perp$ and set theoritically written as

$$W^\perp = \{\mathbf{x} : \quad \mathbf{x}.u = 0 \ \ \forall \ \ u \in W\}$$

16. W is a subspace of R^n.

17. Let A be mXn matrix. Then

$$(RowA)^\perp = NulA, \qquad (ColA)^\perp = NulA^T.$$

18. **Orthogonal Set** A set of vectors in R^n is said to be an orthogonal set if each pair of distinct vectors from the set is orthogonal, that is, $u_i.u_j = 0$ whenever $i \neq j$.

19. Every orthogonal set S of nonzero vectors in R^n is linearly independent and hence forms a basis for the subspace spanned by S.

20. Let $\{u_1, ..., u_n\}$ be an othogonal basis for a subspace W of R^n then each y in W has a unique representation as a linear combination of $u_1, ..., u_n$. In fact if

$$y = c_1 u_1 + ... + c_n u_n.$$

then

$$c_j = \frac{y.u_j}{u_j.u_j} \quad \forall j = 1, 2, ..., n.$$

21. **Orthogonal projection of y along u.**

$$y_u = \frac{y.u}{u.u} u$$

22. **Orthogonal projection of y perpendicular to u.**

$$y_{u^\perp} = y - \frac{y.u}{u.u} u$$

23. **Unit Vectors**

$$\frac{u}{\|u\|}$$

24. A set $\{u_1, ..., u_n\}$ is an orthonormal set if it is an orthogonal set of unit vectors.

25. An mXn matrix U has orthonormal coloumns if and only if $U^T U = I$

26. An nXn matrix U has orthonormal coloumns if and only if $U^T = U^{-1}$

27. **Inner Product** An inner product on a vector space V is a function that, to each pair of vectors u and v in V , associates a real number $< u, v >$ and satisfies the following axioms , for all u,v and w in V and all scalars c:

    (a) $< u, v >=< v, u >$
    (b) $< u + v, w >=< u, w > + < v, w >$
    (c) $< cu, v >= c < u, v >$
    (d) $< u, u > \ \geq \ 0$ and $< u, u >= 0$ if and only if $u = 0$.

    A vector space with an inner product is called an inner product space.

28. Lengths defined in inner product space by

$$\|v\| = \sqrt{<v.v>}\,.$$

29. **The Cauchy-Schwarz Inequality**
Let V be an **inner product** space then

$$|< u, v >| \leq \|u\| \; \|v\|.$$

30. For any inner product space V and for all u and v in V ,

$$\|u + v\| \leq \|u\| + \|v\|.$$