3. (20%)

The manager of computer operations of a large company wishes to study computer usage of two departments within the company, the accounting department and the research department. A random sample of five jobs from the accounting department in the last week and six jobs from the research department in the last week are selected, and the processing time (in seconds) for each job is recorded.

Department	Processing Time (in seconds)	n	$\bar{x}$	s
Accounting	9 3 8 7 12	5	7.8	3.27
Research	4 13 10 9 9 6	6	8.5	3.15

a. Set up a 95% confidence interval for the average processing time for all jobs in the accounting department. Interpret the interval in words.

① $\bar{X} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$; $df = n-1 = 5-1 = 4$

① $7.8 \pm 2.776 \frac{3.27}{\sqrt{5}}$

7.8 ± 4.06

① $\boxed{3.74 \le \mu \le 11.86}$

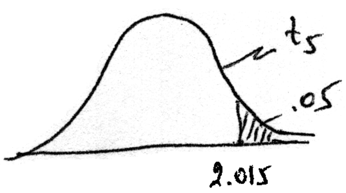
① We are 95% confident that the average processing time for all jobs in the accounting department lies between 3.74 and 11.86 seconds.

b. At $\alpha = 5\%$, is there evidence that the average processing time in the research department is greater than 6 seconds?

1 $H_0: \mu_R \le 6$ sec vs $H_a: \mu_R > 6$ seconds where μ_R = the average processing time in the Research Department

2 $\alpha = 5\% = .05$

3 $t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} = \frac{8.5 - 6}{3.15/\sqrt{6}} = 1.94$ $df = 5$



Reject H_0 if $t \ge 2.015$

① 5 Since $t = 1.94 < 2.015$ we do not have enough evidence to reject H_0 that is we cannot conclude that the average processing time in the research department is greater than 6 seconds.

10 c. Is there evidence of a difference in the mean processing time between the accounting department and the research department? Use $\alpha = 5\%$. Be sure to state Ho, Ha in words as well as any assumptions used.

2).

① H_0: $\mu_A = \mu_R$: The two departments have the same average processing time

H_a: $\mu_A \neq \mu_R$ The two departments have different average processing times

② $\alpha = .05$

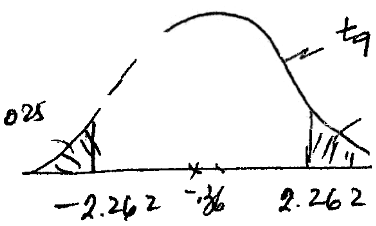
③ $df = 5+6-2 = 9$

$$s_p^2 = \frac{4(3.27)^2 + 5(3.15)^2}{9} = 0.265$$

2 $$t = \frac{(\bar{X}_A - \bar{X}_R) - (\mu_A - \mu_R)}{\sqrt{s_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$t = \frac{(7.8 - 8.5) - 0}{\sqrt{10.265\left(\frac{1}{5} + \frac{1}{6}\right)}} = -.36$$

④ 2



Reject H_0 if $t \leq -2.262$ OR $t \geq 2.262$

⑤ Do not Reject H_0. There is not enough evidence to support that the two departments have different average times. 2

Assumptions: 2
- samples are independent
- Both populations are Normal
- Equal variances.

4. (12%)

The intelligence quotient of a statistics graduate student in the USA has an unknown mean and a known standard deviation equal to 12.

6 a. In a random sample of 144 students the sample mean is found to be equal to 130. Construct a 95% confidence interval for the true population mean.

2 $\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$130 \pm 1.96 \frac{12}{\sqrt{144}}$

130 ± 1.96

2 $\boxed{128.04 \leq \mu \leq 131.96}$

2 We are 95% confident that the true population mean that is the intelligence quotient average, lies between 128.04 and 131.96

b. State any necessary assumption(s) to ensure the validity of the confidence interval in part a.

No assumptions necessary since n is large and σ known

- σ known
- r s

Normal (−2)

c. How large a sample size should do we need in order to construct a 90% confidence interval for the true population mean if we want the margin error not to exceed 3.29?

$$n \geq \left(\frac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2 = \left(\frac{1.645\cdot 12}{3.29}\right)^2 = 36$$

$$\boxed{n = 36}$$

5. (16%)

Golf-course designers are concerned that old courses are becoming obsolete because new equipment enables golfers to hit the ball so far. In effect, golf courses are "shrinking". One designer believes that new courses need to be built with the expectation that players will be able to hit the ball more than 250 yards, on average. Suppose a sample of 135 golfers is tested, and their mean driving distance is 256.3 yards. The population standard deviation is 40.4 yards.

4 a. What are the appropriate null and alternative hypotheses to test the designer's research hypothesis?

2 H_0 $\mu \leq 250$ yards The population mean driving distance is ≤ 250 yards

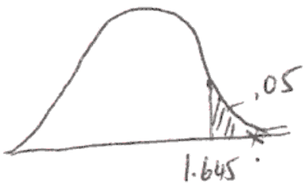
2 H_a: $\mu > 250$ yards > 250 yards

9 b. Test the designer's research hypothesis? Use $\alpha = 5\%$.

Since $n = 135$ is large and $\sigma = 40.4$ yards is known, Use Z-test

$$Z = \frac{\bar{X}-\mu_0}{\sigma/\sqrt{n}} = \frac{256.3-250}{40.4/\sqrt{135}} = 1.81$$

2 Decision Rule: Reject Ho $Z \geq 645$



Reject Ho

Since $Z = 1.81 > 1.645$, we have enough evidence to support the research hypothesis and conclude that the mean driving distance is greater than 250 yards.

3 c. What assumptions are necessary in order to carry out the test?

• Random sample.

6. (16%)

4 a. If an economist wishes to determine whether there is evidence that the average family income in a community exceeds \$25,000, state the null and alternative hypotheses in symbols and words.

$H_0: \mu \leq \$25,000$: The average family income in this community is less or equal to \$25,000.

$H_a: \mu > \$25,000$: the average family income in a this community exceeds \$25,000

4 b. If a test of hypothesis has a Type I error probability of 0.01, we mean what?

This means that the probability of rejecting the Null hypothesis, while in fact Ho is true, is equal to 1%. that is, we reject H_0 1% of the time while Ho is true

4 c. Suppose we wish to test Ho: $\mu \leq 47$ versus Ha: $\mu > 47$. What will result if we conclude that the mean is greater than 47 when its true value is really 52?

we are making a correct decision

4 d. Explain what is meant by the statement, "we are 90% confident that an interval estimate contains μ ".

means that if repeated samples of size n are selected from a population and a 90% confidence intervals are constructed, 90% of all intervals will con the true population mean.

7. (12%)
In a random sample of 1,200 adults interviewed nationwide, only 324 felt that the salaries of certain government officials should be raised.

8 a. Construct a 95% confidence interval for the actual percentage of adults who share that opinion.

$n = 1200$; $\hat{p} = \frac{324}{1200} = .27$; $\hat{q} = .73$

$$\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

2 $$.27 \pm 1.96 \sqrt{\frac{(.27)(.73)}{1200}}$$

$$.27 \pm .025$$

2 $$\boxed{.245 \le p \le .295}$$

2 → We are 95% confident the actual percentage of adults who share that opinion lies between 24.5% and 29.5%.

4 b. With 95% confidence, what can we assert about the maximum error if we use the sample proportion as an estimate of the corresponding true percentage?

$$\text{Max Error} = z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}} = .025 = |\hat{p} - p$$

We can assert with 95% confidence that our error (that is the deviation of the sample proportion from the true population proportion) is at most 2.5%