Growth and patterns in nature

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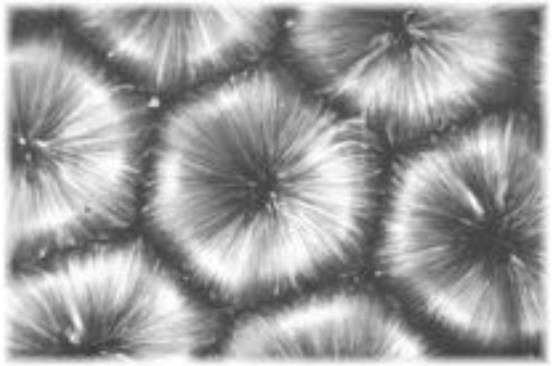
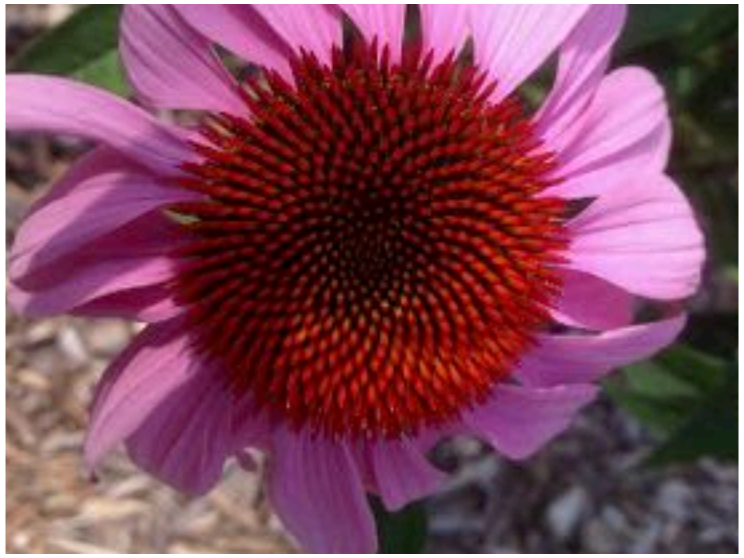
November 19th, 2019

Pattern formation in nature

- How does nature form its structures?





Lets focus on one: CDIMA reaction

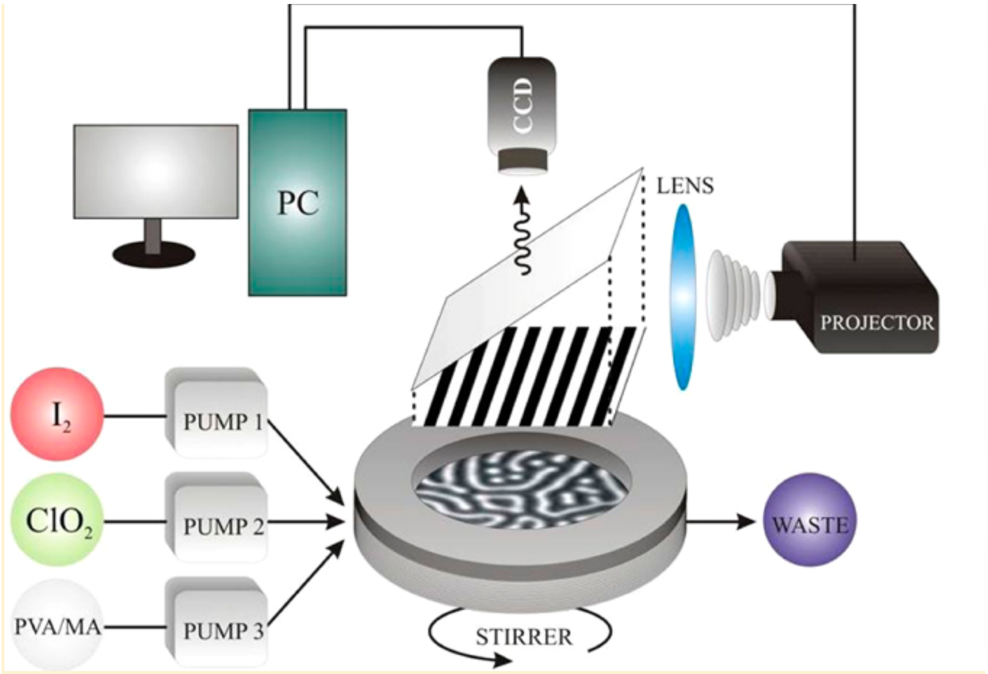
"Simple set of chemical reactions can create spatial patterns"

$MA + I_2 \longrightarrow IMA + I^- + H^+$

$ClO_2 + I \longrightarrow ClO_2^- + 0.5\ I$

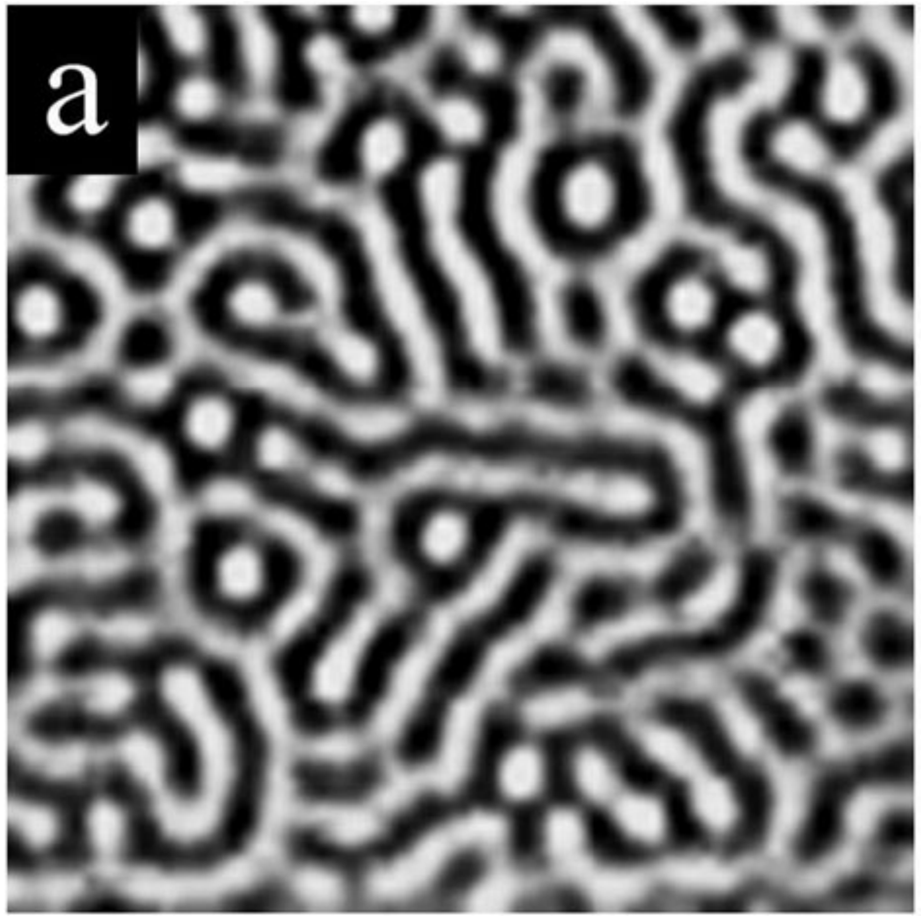
$ClO_2^- + 4I^- + 4H^+ \longrightarrow 2I_2 + Cl^- + 2H_2O$

Well-stirred vat of chemicals feeds into gel suspension, slows down diffusion of chemical species —>



[Epstein '12 et. al]

Chemical reaction occurs in a spatially non-constant manner: Striped and spotted states form at *random orientations.*



$[I^-]$ concentration

How does this happen?

Mathematical modeling of patterns

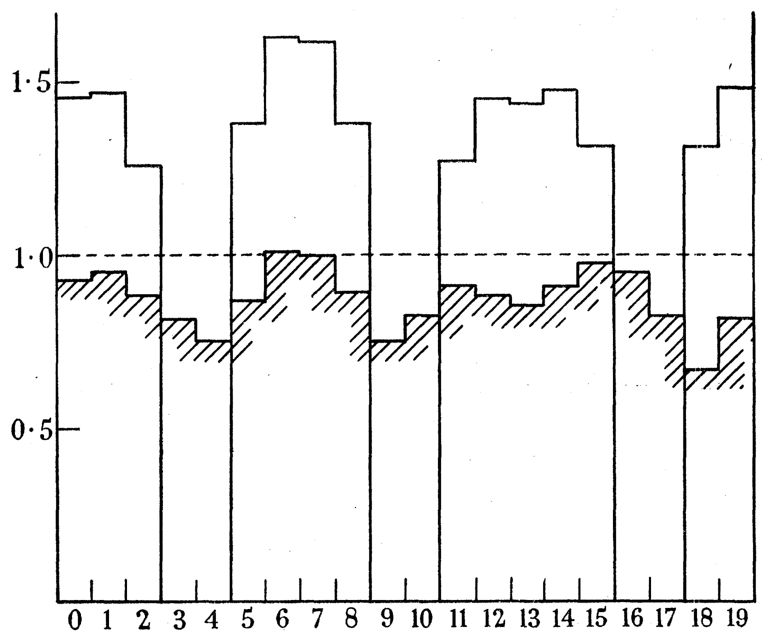
- Alan Turing:

THE CHEMICAL BASIS OF MORPHOGENESIS

By A. M. TURING, F.R.S. *University of Manchester*

(*Received* 9 *November* 1951—*Revised* 15 *March* 1952)

Turing's idea: $\textit{Reaction} + \textit{Diffusion} = \textit{spatial patterns}$



3. Concentrations of Y in the development of the first specimen (taken from table 1). ---- original homogeneous equilibrium; ////// incipient pattern; ——— final equilibrium.

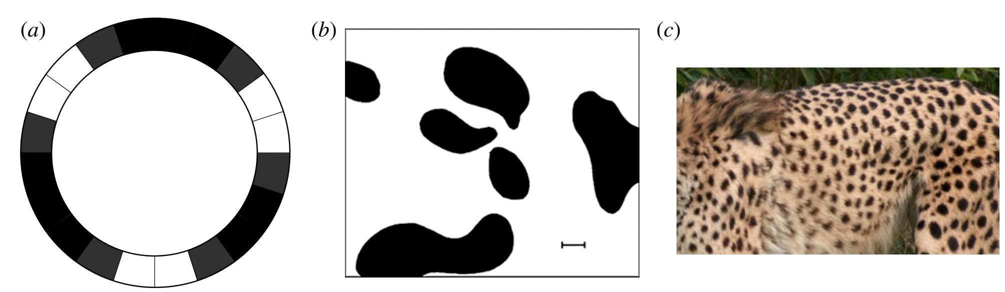


Figure 2. (*a*) The morphogen pattern in a ring of cells as deduced by Turing. The greyscale indicates concentration differences. (*b*) Turing's hand-calculated 'dappled pattern' created by a morphogen scheme in two dimensions [1, fig. 2]. (*c*) The resemblance to animal markings (here a cheetah) was obvious, albeit at this point no more than qualitative.

- *Mathematics reveals how simple physical mechanisms can combine and lead to complicated spatial behavior*
  - *Foundational work inspiring whole area(s) of research*

Chemical reaction

- Lets start with no spatial variation in the model: (ordinary differential equation)

$$u_t = a - bu - \frac{4uv}{1+u^2} - \phi \qquad \Longrightarrow U_t = F(U) = \begin{pmatrix} f(u,v) \\ g(u,v) \end{pmatrix} \qquad U = \begin{pmatrix} u \\ v \end{pmatrix} \begin{matrix} \text{- concentration of } [I^-] \\ \text{- concentration of } [ClO_2^-] \end{matrix}$$

$$v_t = \sigma\left(bu - \frac{uv}{1+u^2} + \phi\right)$$

$a, b, \phi, \sigma \rightarrow$ parameters

- In a well-stirred mixture there exists a chemical equilibrium:

$$F(U_*) = 0 \qquad \Longrightarrow \; U_t = 0$$

- Dynamics near equilibrium: Taylor expand about $U = U_*$

$$V_t = F(U_* + V) = F(U_*) + DF(U_*)V + O(|V|^2)$$

- 2D-Linear system: $V_t = AV$, $\qquad A = DF(U_*) = \begin{pmatrix} \partial_u f & \partial_v f \\ \partial_u g & \partial_v g \end{pmatrix}$

    - Stability: Do solutions $V(t)$ grow or decay in time?

Linear system

- Do solutions $V(t)$ grow or decay in time?

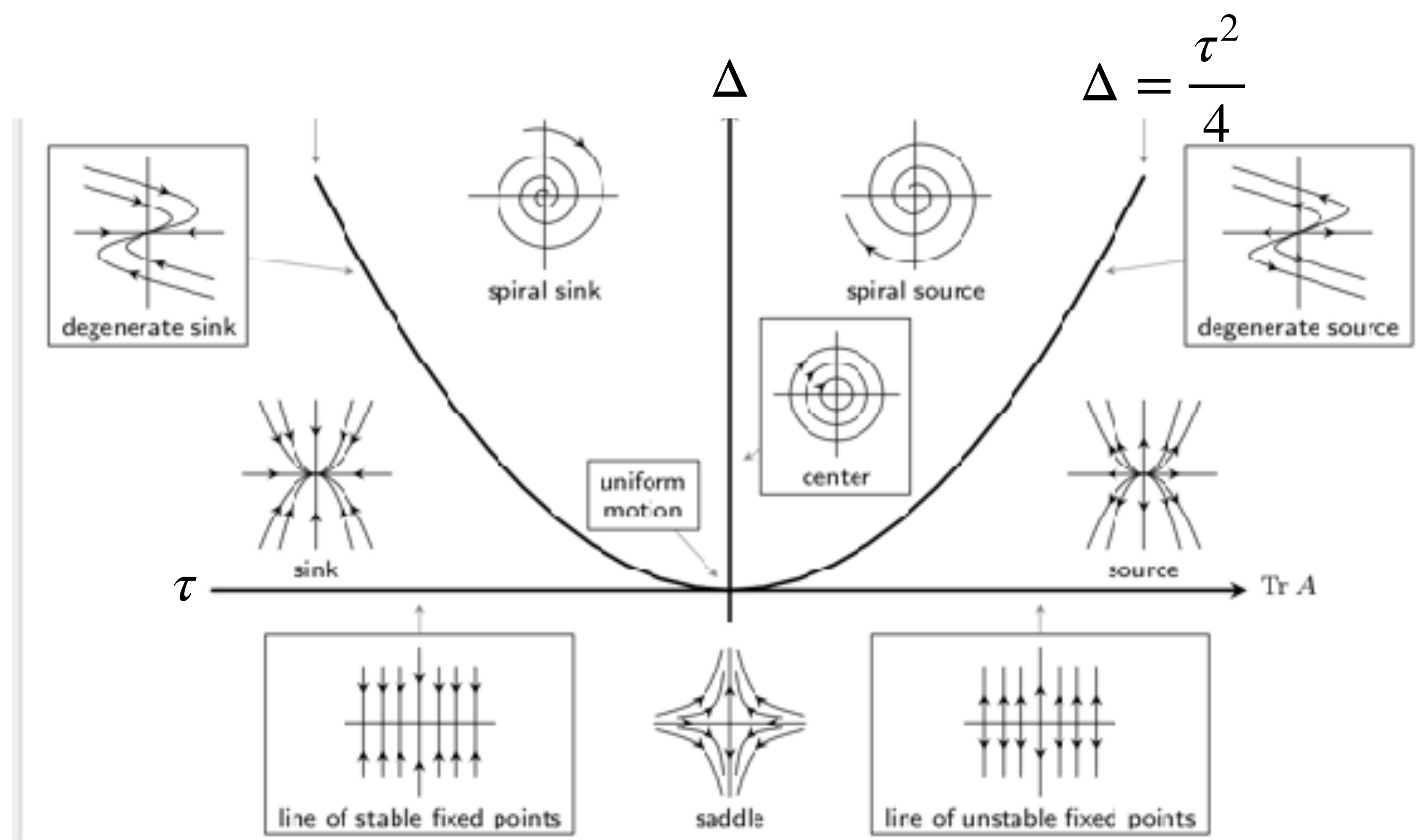
$$V_t = AV, \qquad A = DF(U_*) = \begin{pmatrix} \partial_u f & \partial_v f \\ \partial_u g & \partial_v g \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Study the eigenvalues of A: $\lambda_{\pm} = \dfrac{\tau \pm \sqrt{\tau^2 - 4\delta}}{2}$, $\qquad \tau = a_{11} + a_{22},\ \delta = a_{11}a_{22} - a_{12}a_{21}$

Stability condition:

$$\operatorname{Re}\lambda_{\pm} < 0 \iff \tau < 0,\ \delta > 0$$

$$\implies V(t) \text{ decays in time}$$



Upshot: If well-stirred, chemical concentrations will approach a stable equilibrium state.

Reaction - Diffusion

- Now allow the chemical species to spread or *diffuse* in space (i.e. fed them into a gel):

$$u_t = d_u u_{xx} + a - bu - \frac{4uv}{1+u^2} - \phi$$

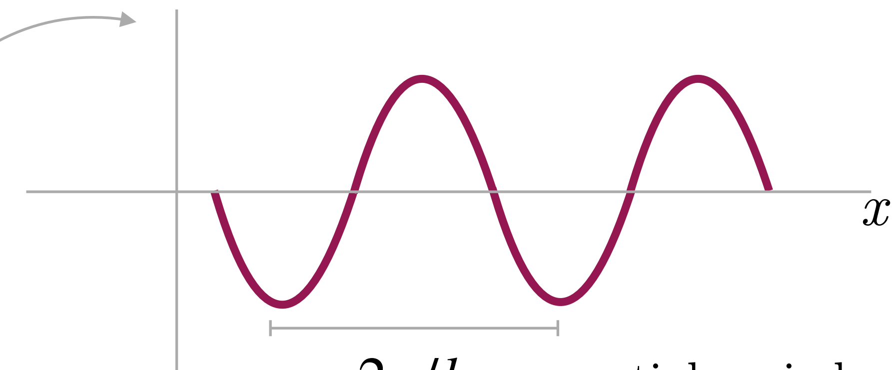
(partial differential equation)

$$v_t = d_v v_{xx} + \sigma(bu - \frac{uv}{1+u^2} + \phi)$$

- What effect does the diffusion have?

Spatial "mode"

$\mathrm{Re}\, e^{ikx} = \cos(kx)$



$2\pi/k$ = spatial period

- Linear stability analysis about U_*: insert $U = U_* + V_k e^{ikx}$

- Taylor expand and truncate at linear order in V:

$$\frac{d}{dt}V_k = A(k)V_k, \qquad A(k) := \begin{pmatrix} -d_u k^2 & 0 \\ 0 & -d_v k^2 \end{pmatrix} + DF(U_*)$$

$$DF(U_*) = \begin{pmatrix} \partial_u f & \partial_v f \\ \partial_u g & \partial_v g \end{pmatrix}$$

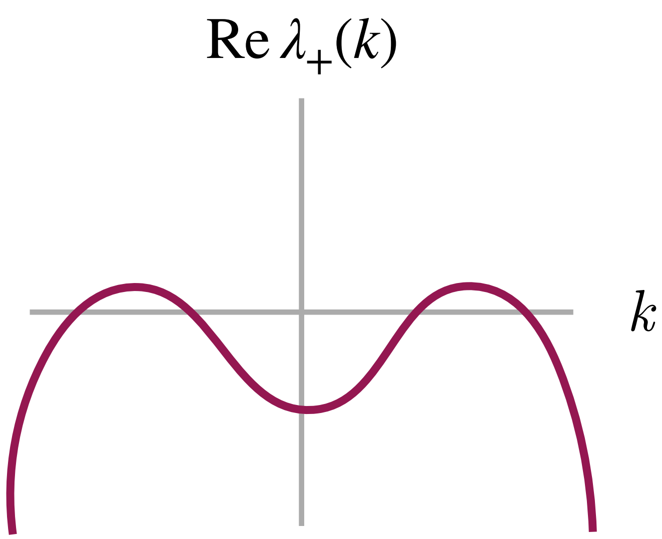
Eigenvalues: $\lambda_\pm(k) = \tau(k) \pm \sqrt{\tau(k)^2 - 4\delta(k)}$,

$$\tau(k) = a_{11} + a_{22} - k^2(d_u + d_v),$$

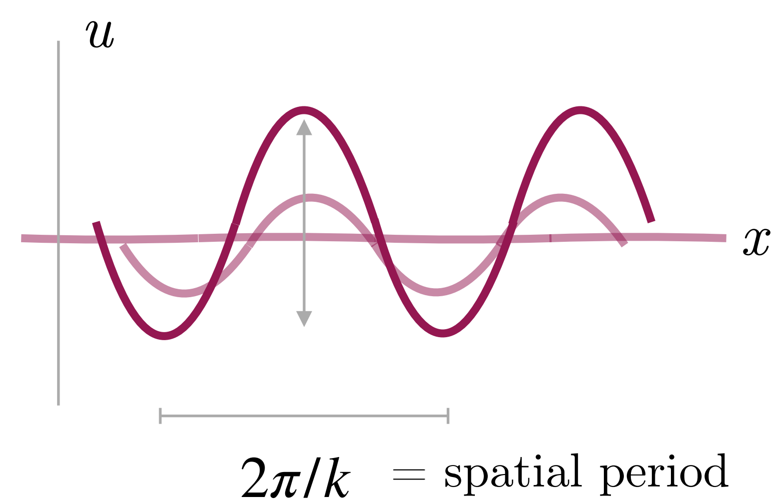
$$\delta(k) = (a_{11} - d_1 k^2)(a_{22} - d_2 k^2) - a_{12}a_{21}$$

"Turing" instability

- Choose parameters correctly: typically $d_u \ll 1$ and $d_v \sim 1$ (short range activation, long range inhibition), obtain an unstable eigenvalue $\lambda_+(k)$ with non-zero wavenumber



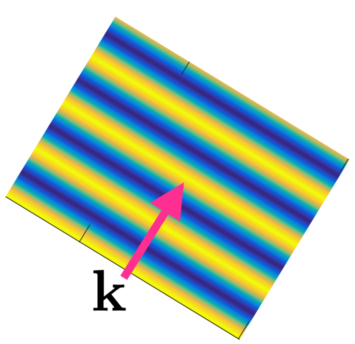
$\Longrightarrow$

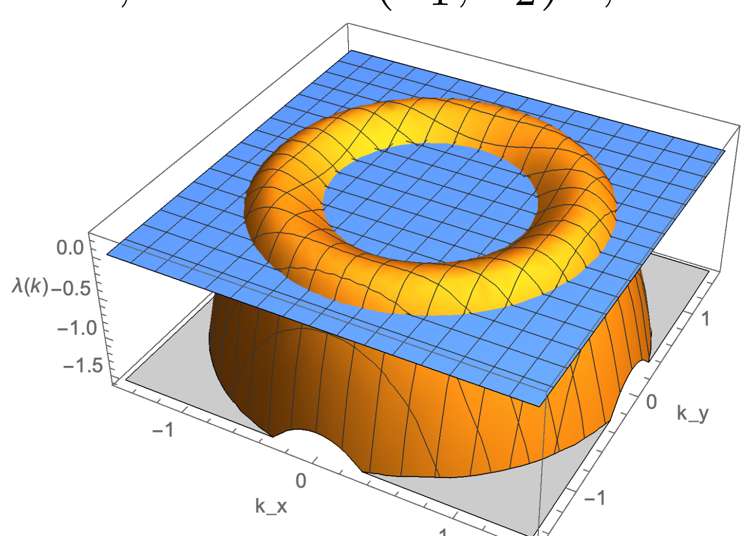


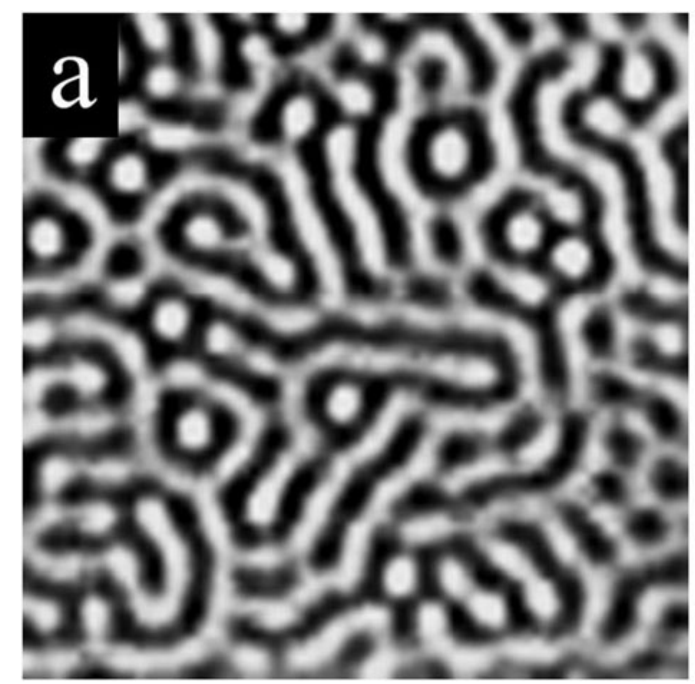
"Linear growth of spatially non-constant modes"

In two spatial dimensions, this corresponds to an annulus of wavenumbers becoming unstable

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \qquad e^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}, \qquad \mathbf{x} = (x_1, x_2)^T, \qquad \mathbf{k} = (k_x, k_y)^T, k^2 = k_x^2 + k_y^2$$







"Turing Pattern": diffusion induced, spatially periodic equilibrium

If no diffusion, then no spatial patterns!

Light-sensitivity of CDIMA reaction

- Light suppresses spatial patterns
- Traveling mask speed *selects* pattern and *mediates* defects
- Experimental model for a growing organism

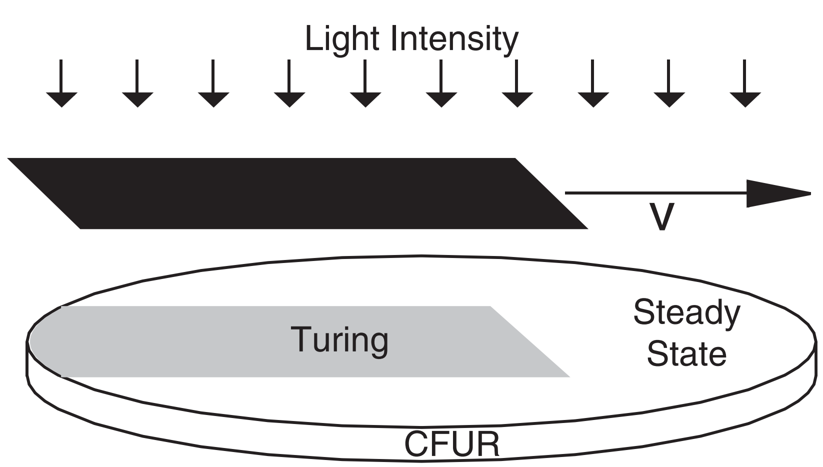
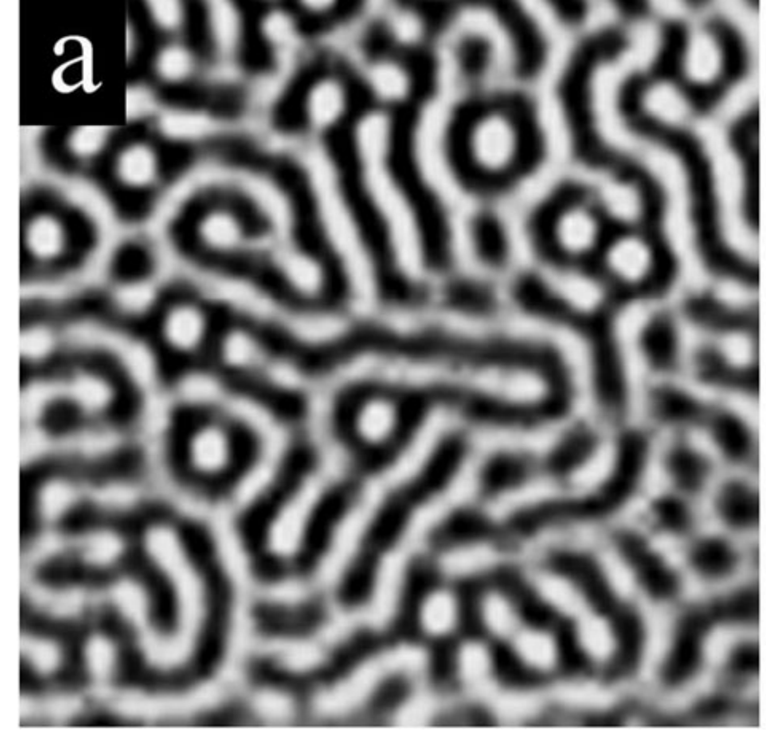
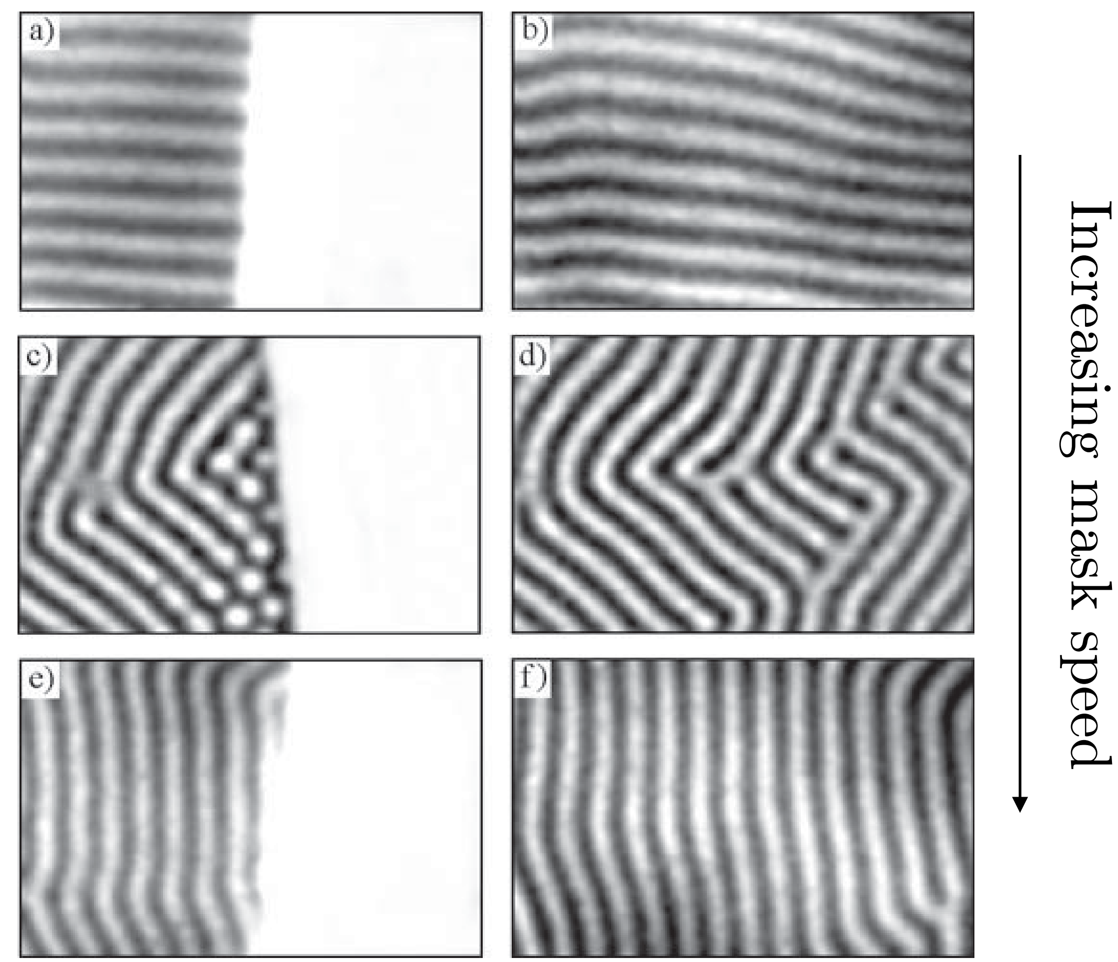


FIG. 1. Schematic of the experiment. A moving opaque mask image creates a growing shadow domain where Turing patterns can develop. In the illuminated domain the pattern is suppressed.



Homogeneous medium, [Epstein et. al. '12]

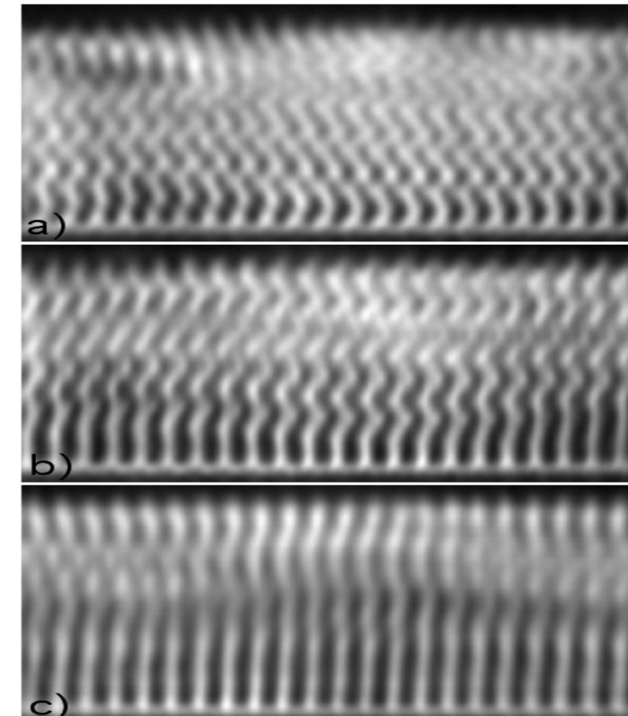


Heterogeneous medium, [Miguez et. al. '06]

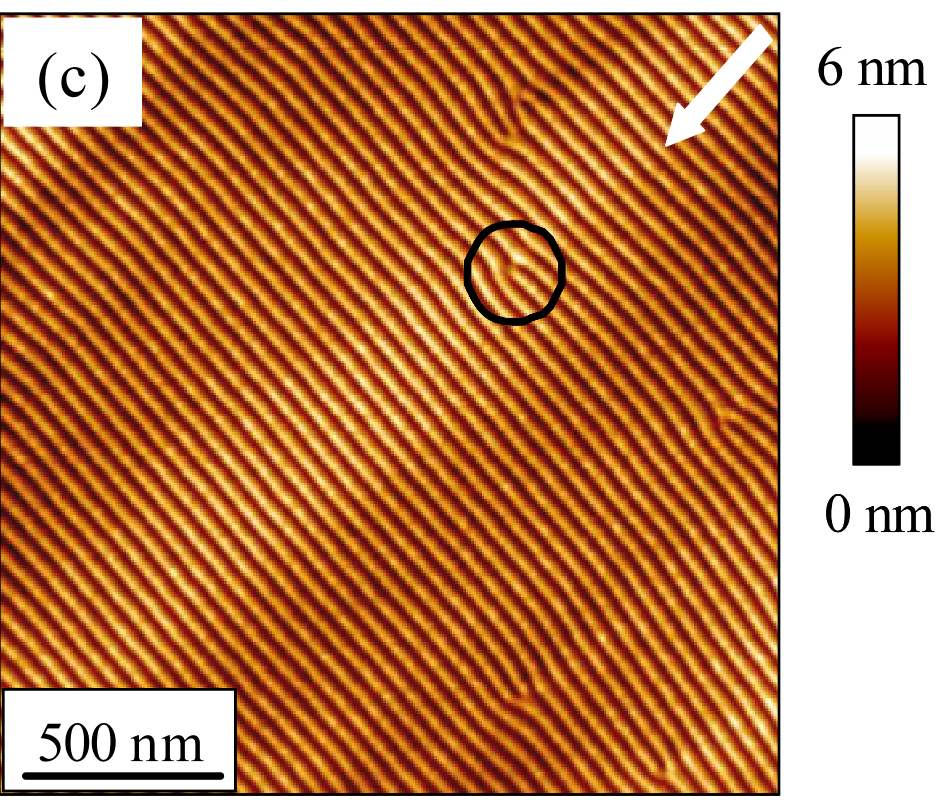
Patterns and Growth

How does growth or spatial heterogeneity mediate or select patterns?

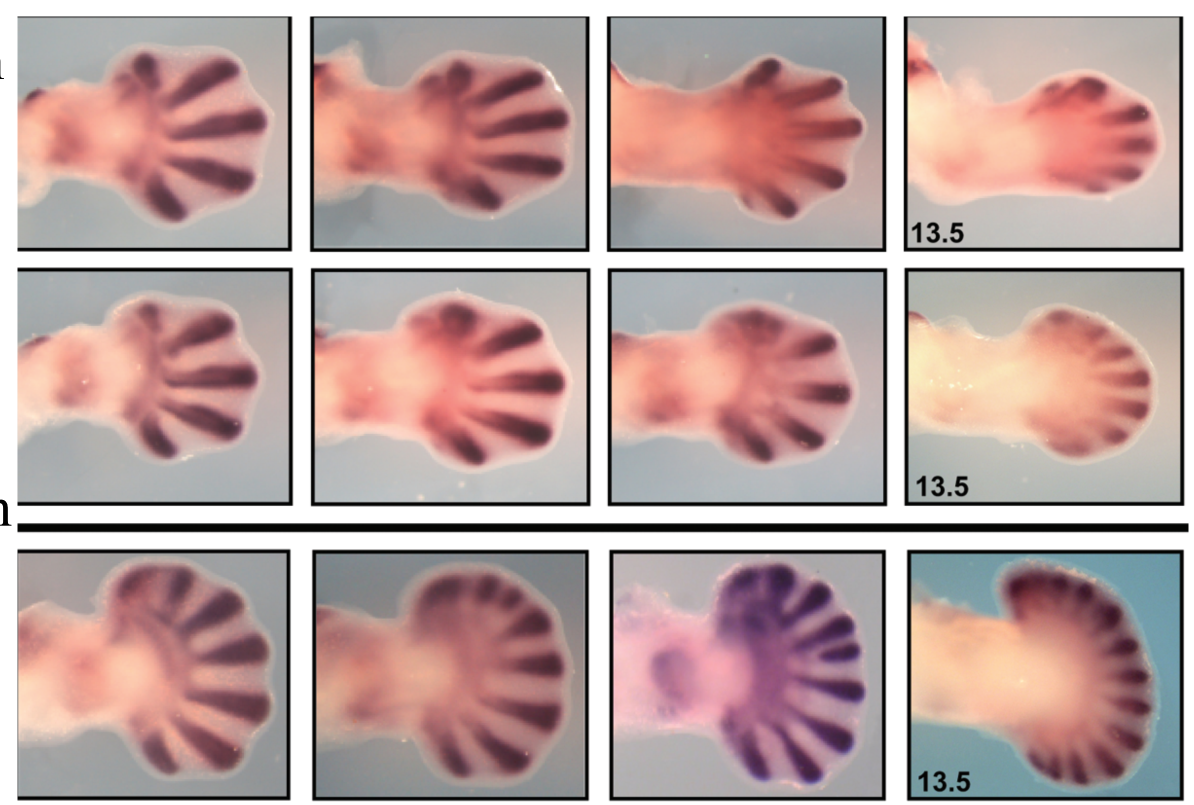
- External mechanism travels through system, or system domain grows, mediating pattern formation.
- Aim is to more efficiently and effectively form novel materials at various length-scales, and understand growth processes in nature.



Quenching/solidifcation in Eutectic Lamellar Crystals



Ion bombardment of alloys



Early stage digit-patterning

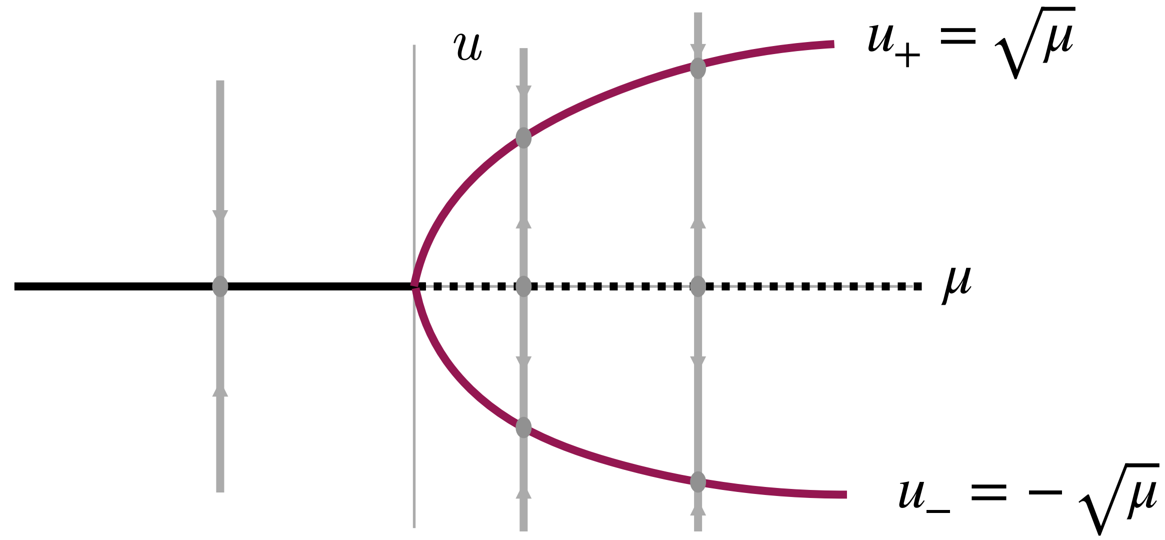
Chemical precipitation

Nonlinear Dynamics viewpoint

Existence, Stability, Bifurcation, and Dynamics of nonlinear coherent states, which organize behavior

- Simple ODE example: $u_t = \mu u - u^3$ $\quad u \in \mathbb{R}, \mu$ - bifurcation parameter

$u_* = 0$ equilibrium, changes stability at $\mu = 0$



- *Bifurcation indicated by a linear instability of* $u_* = 0$*, at* $\mu = 0$*:* $\quad v_t = \mu v$
- Dynamics saturated by nonlinearities, non-trivial states $u_\pm$ attract nearby trajectories
- What is the "basin of attraction" of each non-trivial state $u_\pm$?

Dynamics of spatial patterns?

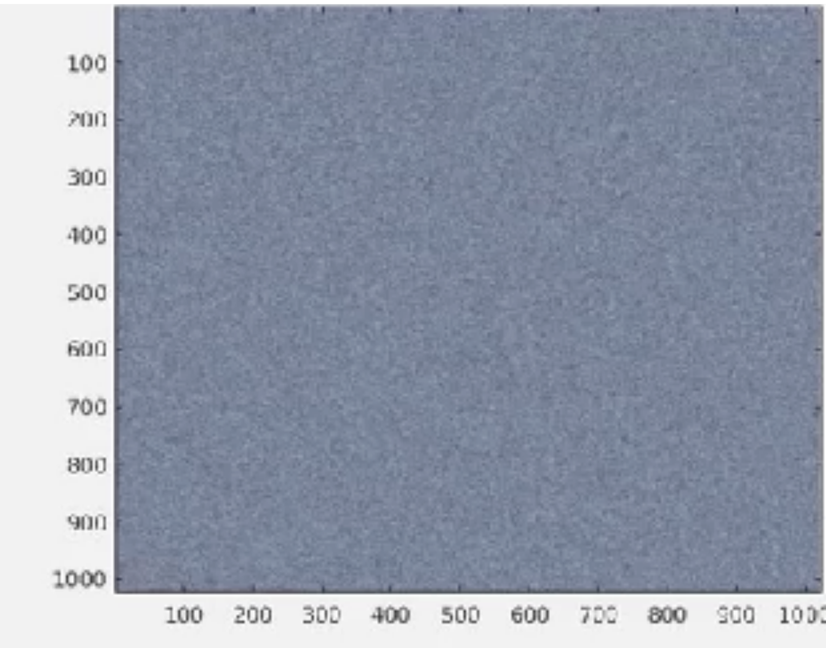
Can we study patterns in a simpler setting compared with coupled system: so a ***scalar*** equation?

Could introduce (2D) spatial dependence by adding in diffusion:

$$u_t = \Delta u + \mu_0 u - u^3, \qquad \Delta := \partial_x^2 + \partial_y^2$$

"Diffusion" "Reaction"

Allen-Cahn equation

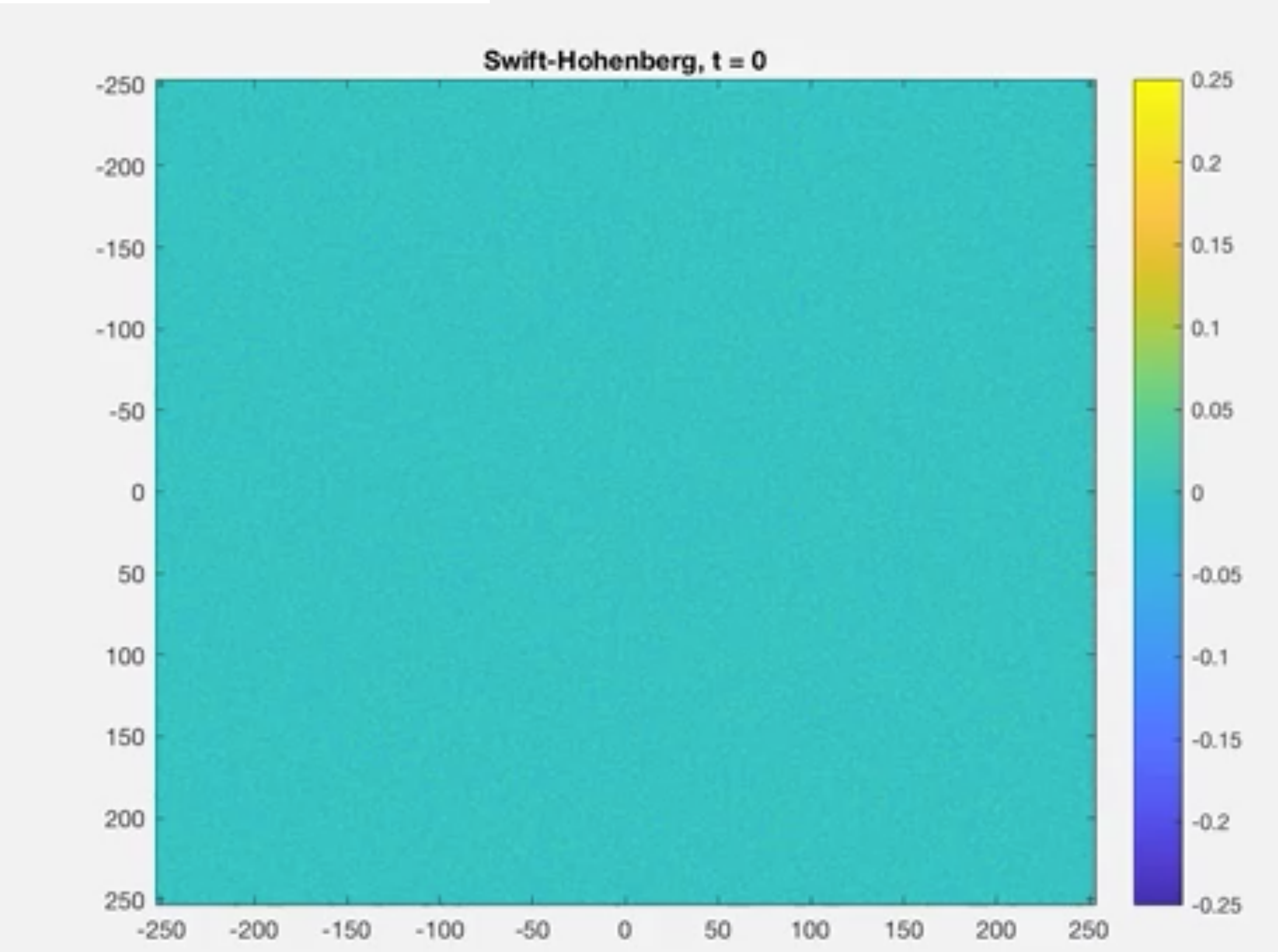


but patterned solutions are unstable, or not persistent

Turns out... a simple way to get stable patterns, in scalar PDE:

$$u_t = -(1+\Delta)^2 u + \mu_0 u - u^3,$$

Swift-Hohenberg equation



[SH-'77], [Cross, Hohenberg '93]

Pattern forming model: The Swift-Hohenberg equation

[SH-'77], [Cross, Hohenberg '93]

$$u_t = -(1+\Delta)^2 u + \mu_0 u - u^3, \quad u : \mathbb{R}^n \to \mathbb{R},$$

- u - order parameter, measures state of system
- μ_0 -bifurcation/"onset" parameter: $u \equiv 0$ stable/unstable for $\mu_0 \lessgtr 0$

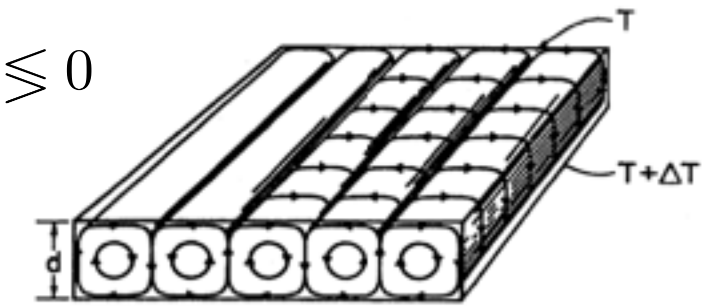
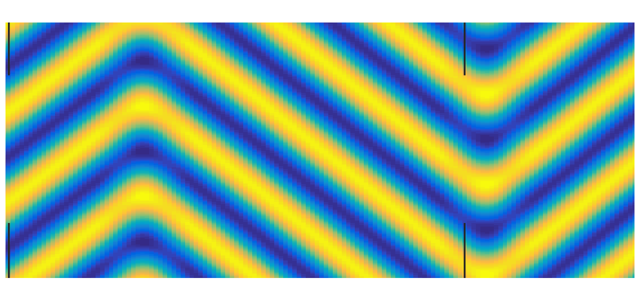


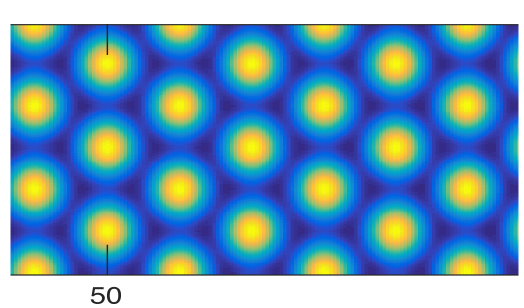
FIG. 1. Schematic picture of Rayleigh-Bénard convection showing fluid streamlines in an ideal roll state.

- Originally derived for Rayleigh-Bénard convection —>
- Universal model for many phenomena:
  - In fact Turing was working on a similar equation! [Dawes '15]
    - "*Outline of development of the Daisy*" [Turing, unfinished draft]
  - Been used as a model for liquid crystals, soft-materials, plant phylotaxis, reaction-diffusion
- Nice starting point because much is rigorously known:

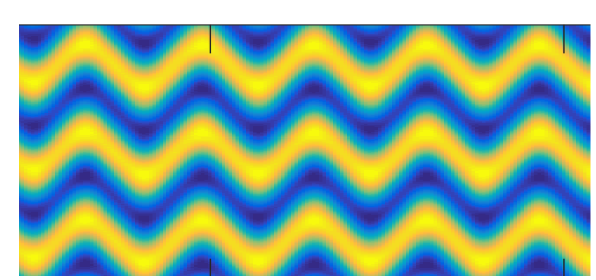
Grain Boundaries



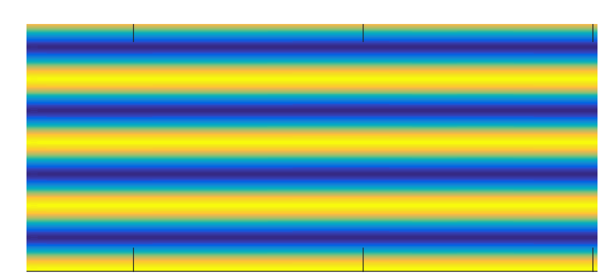
Hexagons



Zig-Zags



Stripes



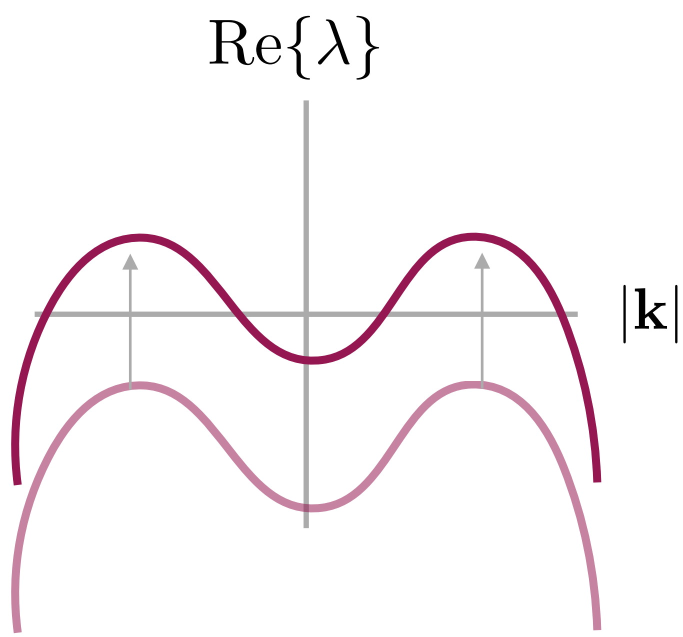
Patterns in Swift-Hohenberg equation

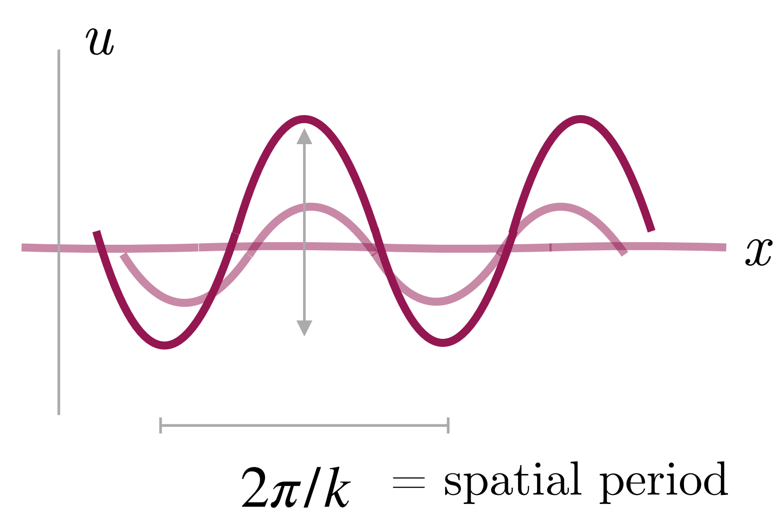
$$u_t = -(1+\Delta)^2 u + \mu_0 u - u^3, \quad u : \mathbb{R}^n \to \mathbb{R},$$

- **Turing Patterns:** "Pitchfork" bifurcation of a family of spatially periodic equilibria

Turing instability: insert $u = re^{\mathbf{ik}\cdot\mathbf{x}+\lambda t}$ into linear equation yields $\lambda = -(1-k^2)^2 + \mu_0, \quad k = |\mathbf{k}|$

$\mu_0 \nearrow 0$





Linear instability of base state $u = 0 \quad \Longrightarrow$

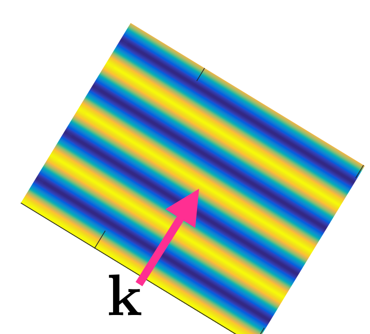
Nonlinear bifurcation of family of stable "roll"/stripe equilibrium states

$$u_p(x) = \sqrt{4(\mu_0 - \kappa)/3}\cos(kx) + \mathcal{O}(|\mu_0 - \kappa|^{3/2}),$$

$$\kappa = k^2 - 1, k \sim 1$$

Rotational invariance -> all orientations of stripes are solutions

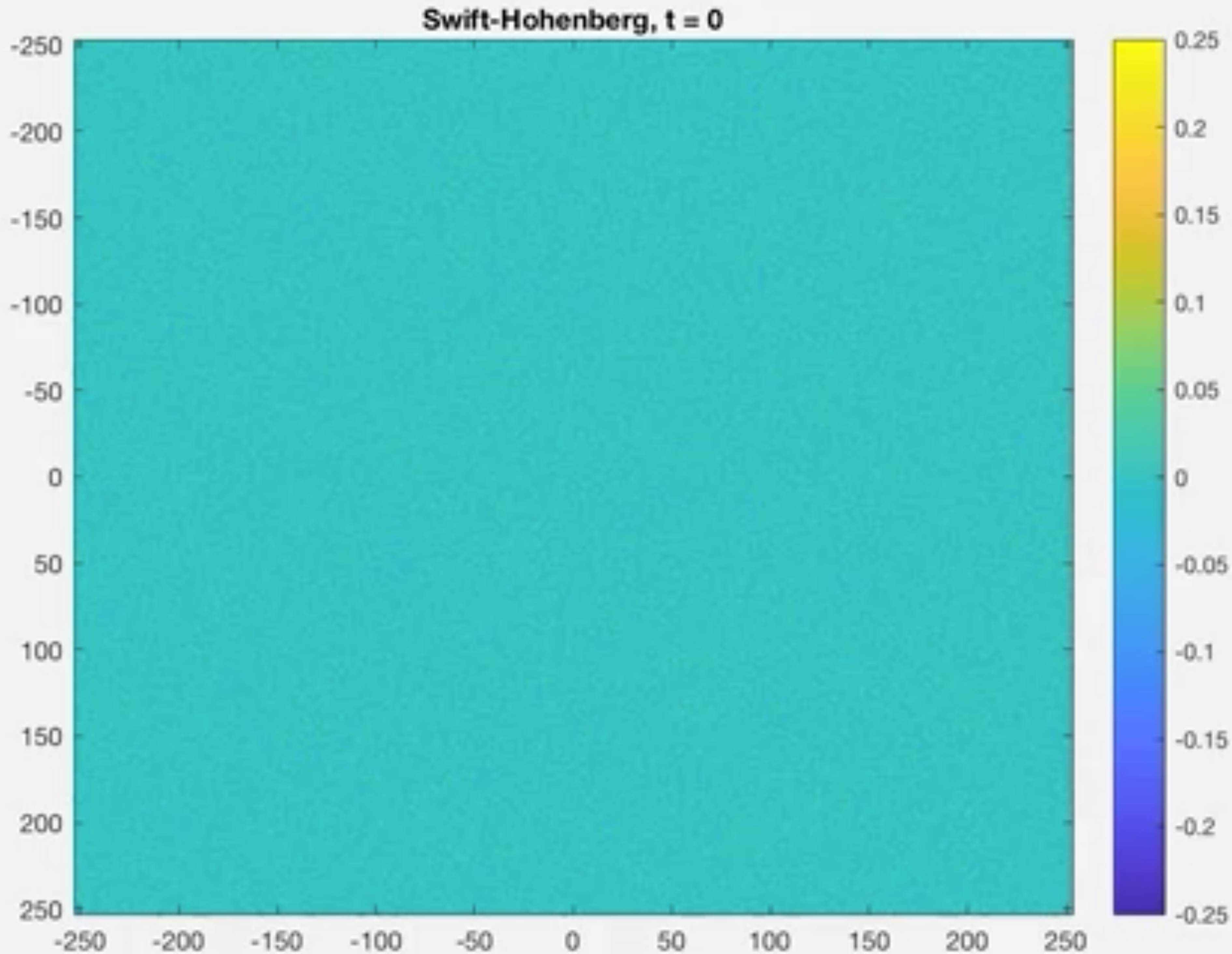
$$u_p(\mathbf{k}\cdot\mathbf{x}; k)$$



[CrossHohenberg'93]

Swift-Hohenberg equation

$$u_t = -(1+\Delta)^2 u + \mu_0 u - u^3, \quad u : \mathbb{R}^2 \to \mathbb{R},$$



"Incoherent patches of patterns"

Growth model in Swift-Hohenberg equation

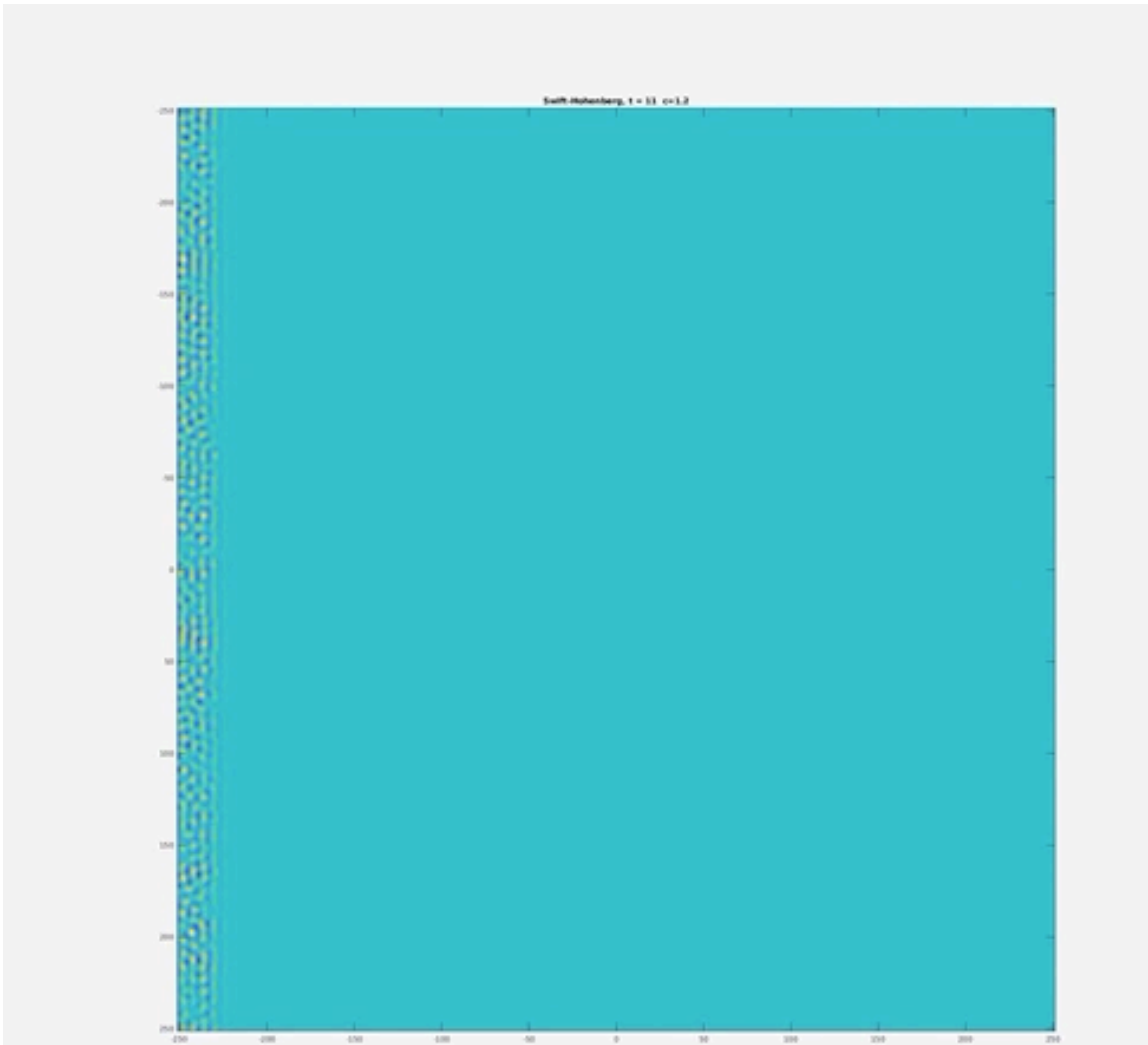
- Spatially progressive bifurcation: jump heterogeneity changes stability of u=0 for $x - ct \gtrless 0$

$$u_t = -(1+\Delta)^2 u + \mu(x - ct)u - u^3, \quad \mu(\xi) = -\mu_0 \mathrm{sgn}(\xi)$$

“Fast” growth

“mild” growth

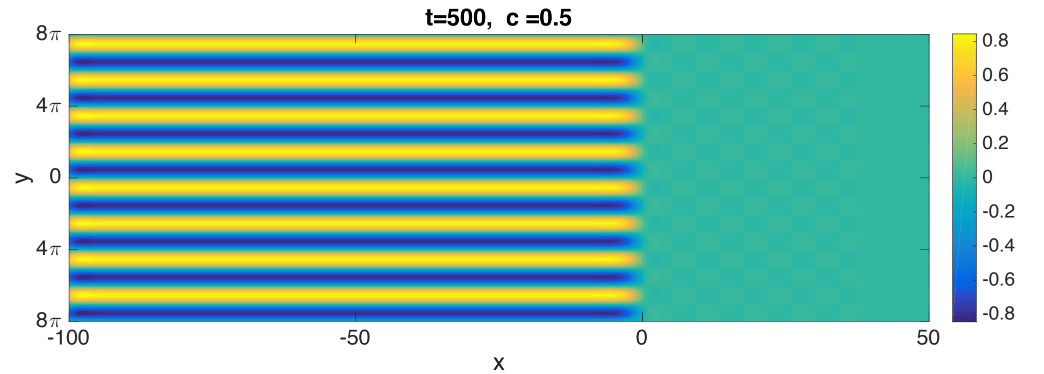
“slow” growth



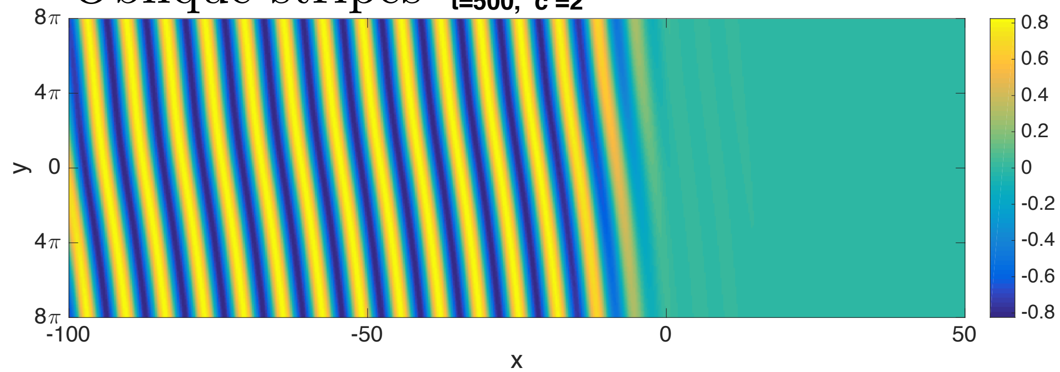
- Similar behavior to experimental RD system

Swift-Hohenberg

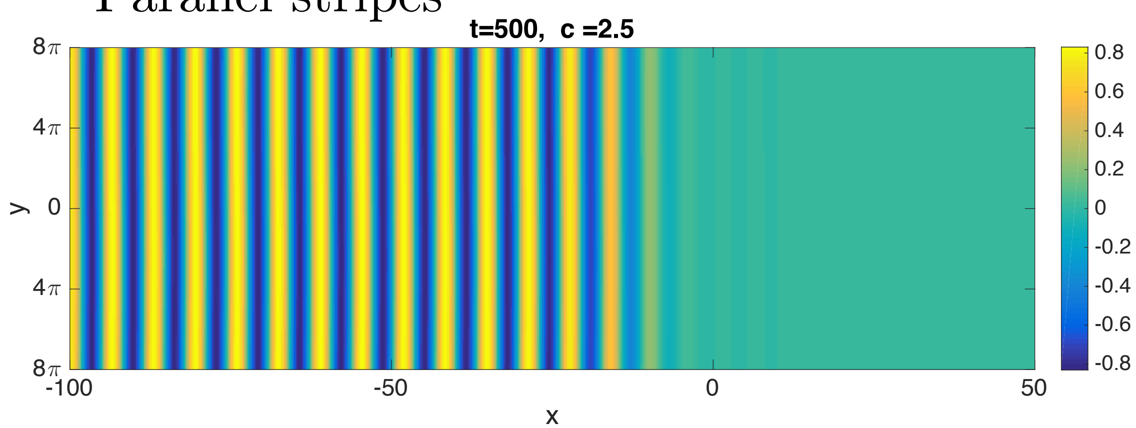
Perpendicular stripes



Oblique stripes

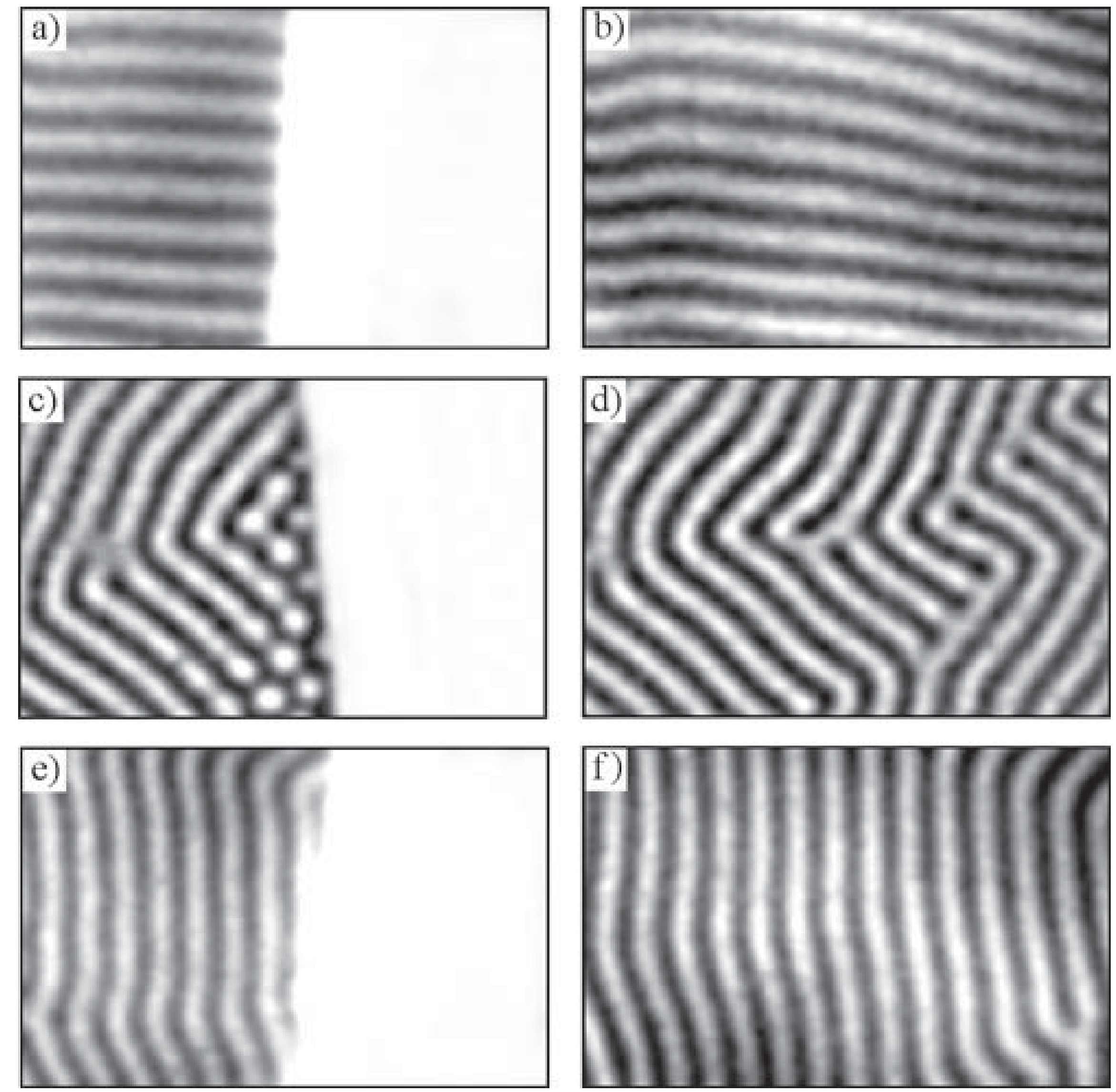


Parallel stripes



Light Sensing RD system

Increasing growth speed c



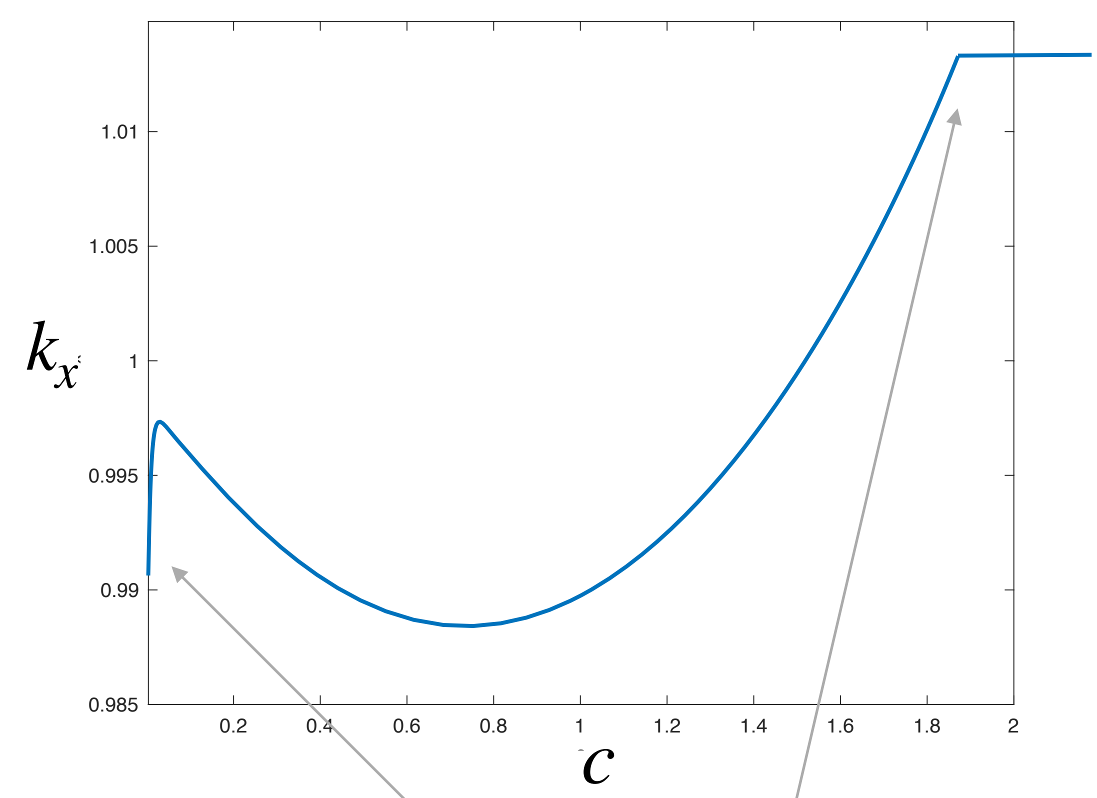
1-D patterns in Swift-Hohenberg

$$u_t = -(1+\partial_x^2)^2 u + \mu(x-ct)u - u^3 \qquad \mu(\xi) = \mu_0 \mathrm{sgn}(-\xi)$$

Study existence of pattern-forming fronts, characterize wavenumber dependence on growth speed c

Wavelength $= \dfrac{2\pi}{k_x}$

Wavenumber "selection" curves

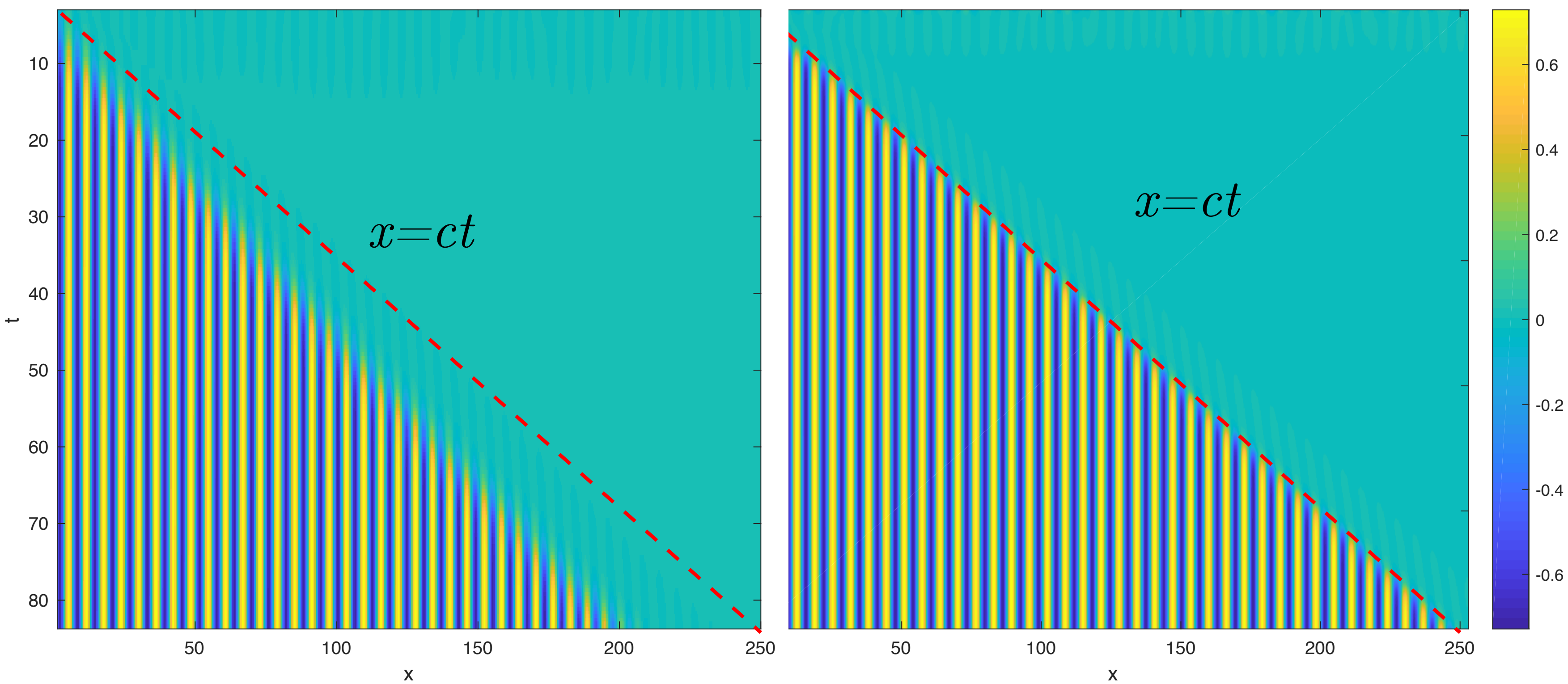


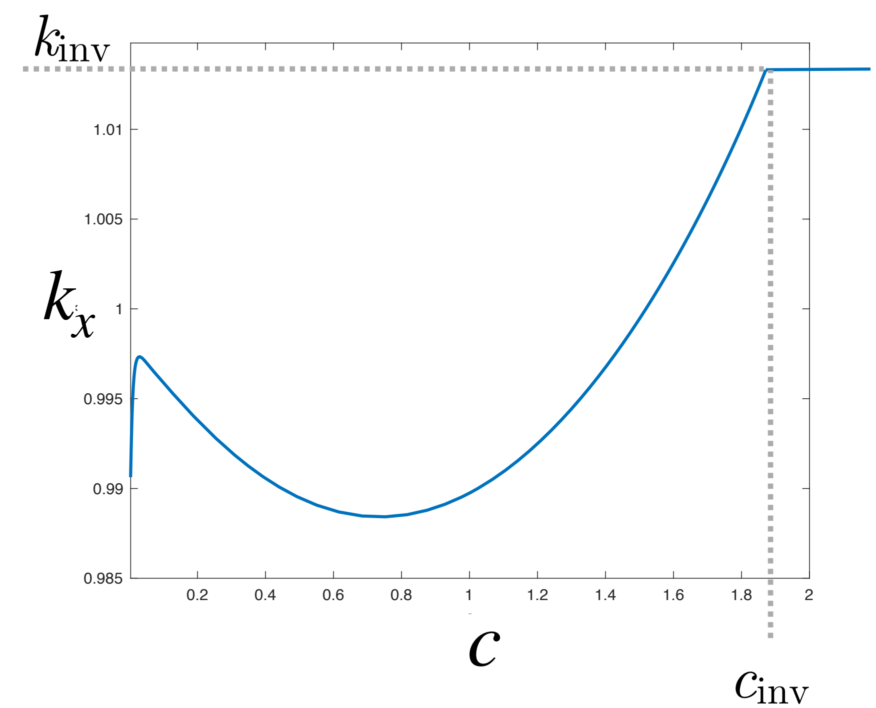
Results for slow and fast speeds

Curves $k_x(c)$ give mechanism and prescription for control of pattern formation process

Fast speeds

$$u_t = -(1+\partial_x^2)^2 u + \mu(x-ct)u - u^3 \qquad \mu(\xi) = \mu_0 \mathrm{sgn}(-\xi)$$





$c > c_{\rm inv}$: pattern selected by unstable homogeneous state behind inhomogeneity.

$c < c_{\rm inv}$: pattern wants to invade faster than you're letting it

Interface has no effect on pattern for $c > c_{\rm inv}$

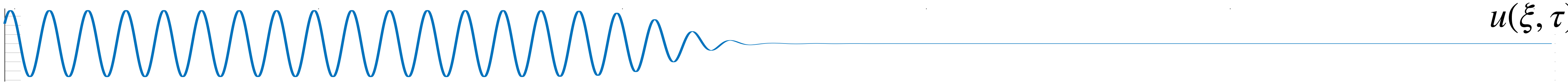
Co-moving frame: $\xi = x - ct$

Fast speeds:

$$\omega u_\tau = -(1+\partial_\xi)^2 u + \mu(\xi)u - u^3 + c\partial_\xi u \qquad (\text{MTW},\ k_y{=}0) \qquad \xi = x - ct$$

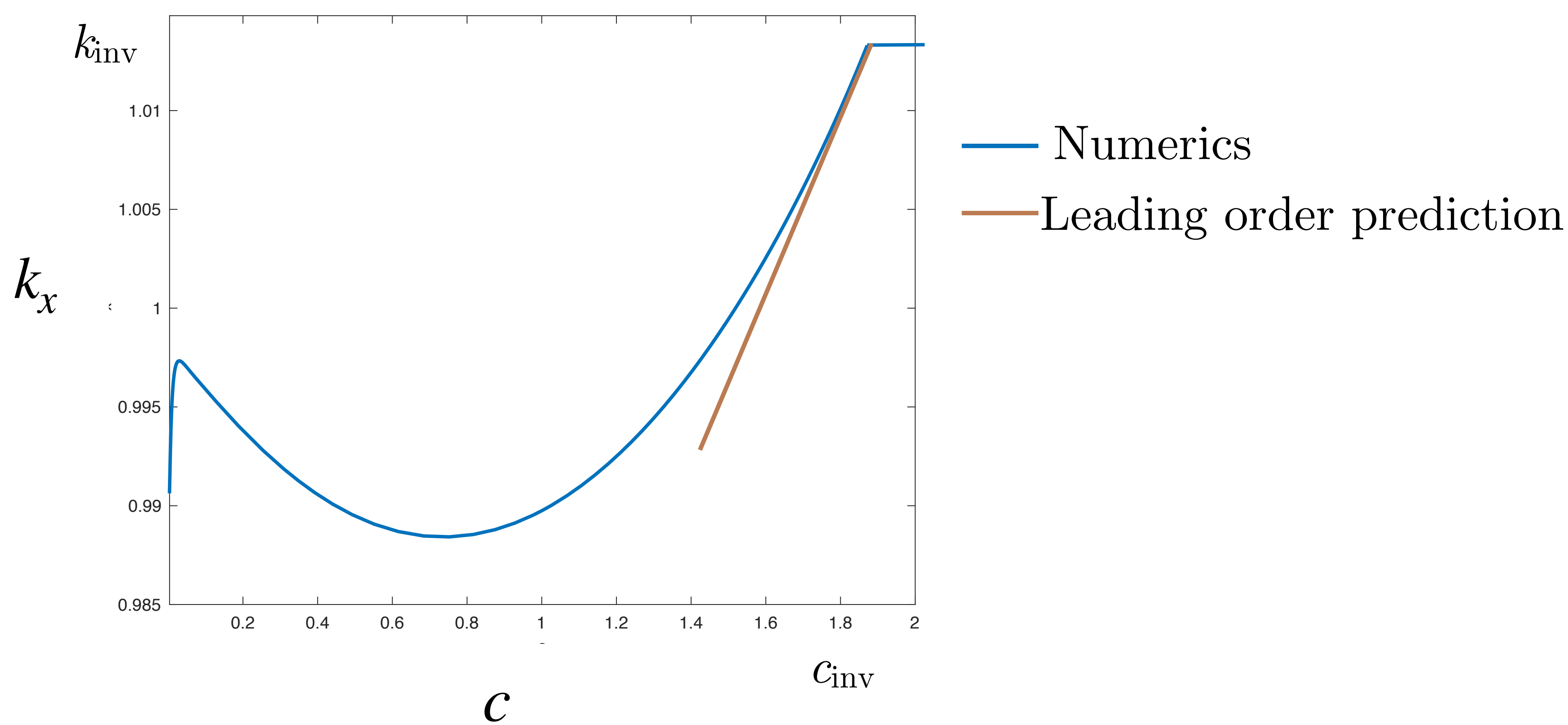
Existence for speeds near detachment point $c \sim c_{\text{inv}} = 4\sqrt{\mu_0}$ $\quad \xi = x - ct$

Look for "small amplitude" solutions with onset multiple scaling: $\quad \mu_0 = \epsilon^2, c = \epsilon\tilde{c}, 0 < \epsilon \ll 1$



Theorem: For ϵ and $4 - \tilde{c} > 0$ *sufficiently small, there exists a pattern forming front with wavenumber*

$$k_x(c) = 1 + \tilde{\gamma}c + \mathcal{O}(c^2)$$



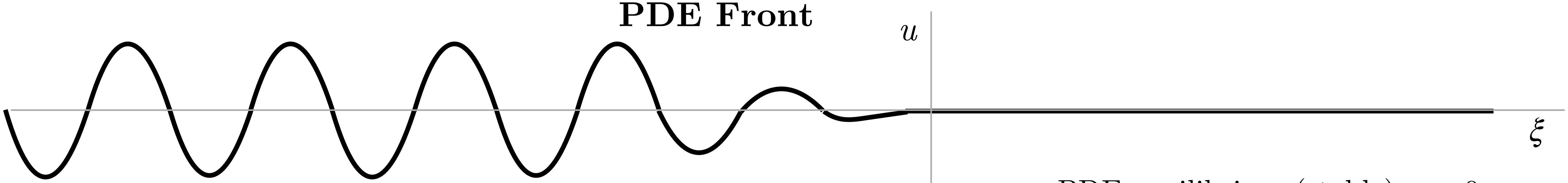
Spatial Dynamics approach

$$\omega u_\tau = -(1+\partial_\xi^2)^2 u + \mu(\xi)u - u^3 + c u_\xi \qquad \xi = x - ct, \tau = \omega t$$

- Look for front solutions as trajectories in a dynamical system with space ξ as evolution variable, in phase space of τ - periodic functions,
- non-autonomous in ξ (but *only* piece-wise constant!)
- Write as first order system —>

$$\begin{aligned} u_\xi &= v, \\ v_\xi &= w, \\ w_\xi &= \theta, \\ \theta_\xi &= (\mu(\xi) - \omega\partial_\tau)u + cv - w - u^3, \end{aligned}$$

PDE Front

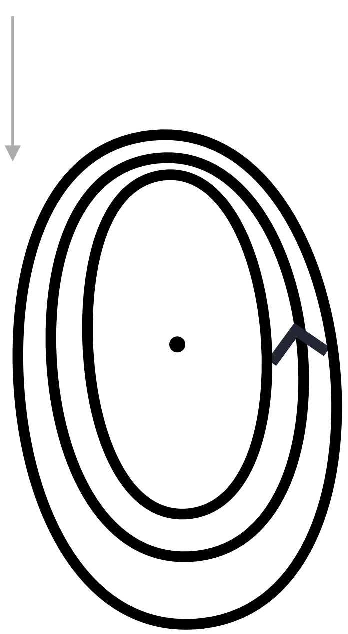


stripe patterns $u_p(k_x\xi + t)$

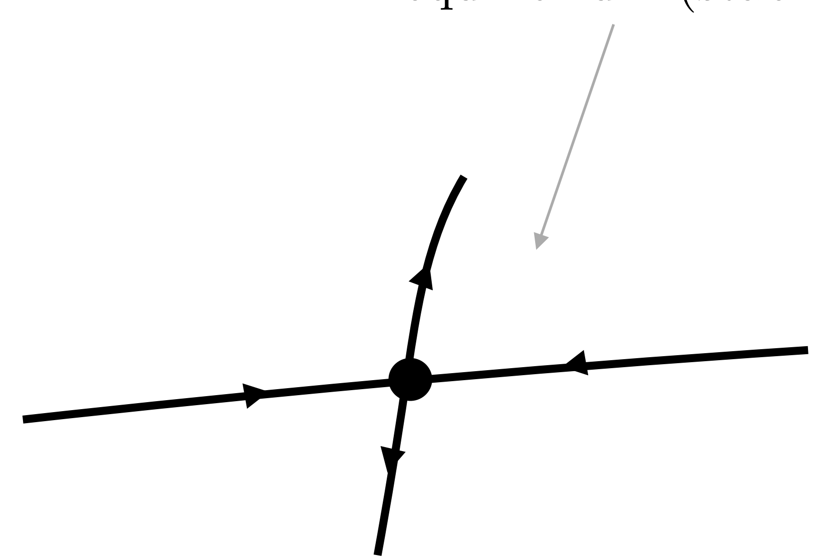
PDE equilibrium (stable) $u \equiv 0$

ODE objects

periodic orbits



$\mu = \mu_0$
$\xi < 0$

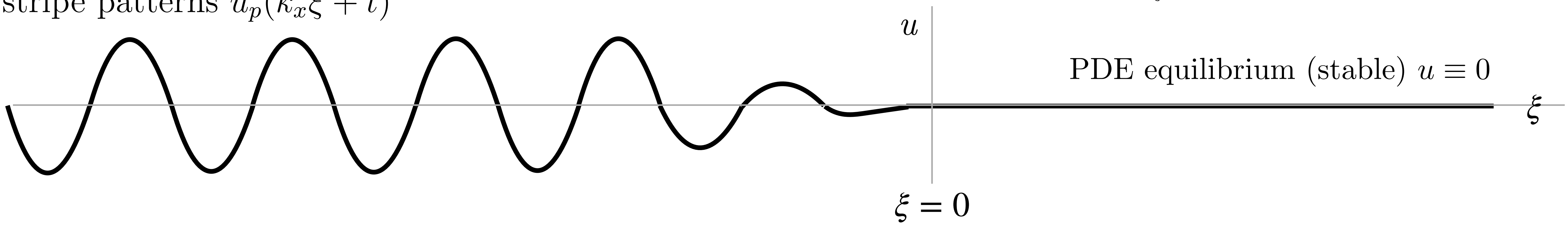


Equilibrium $(u, v, w, \theta) = (0,0,0,0)$

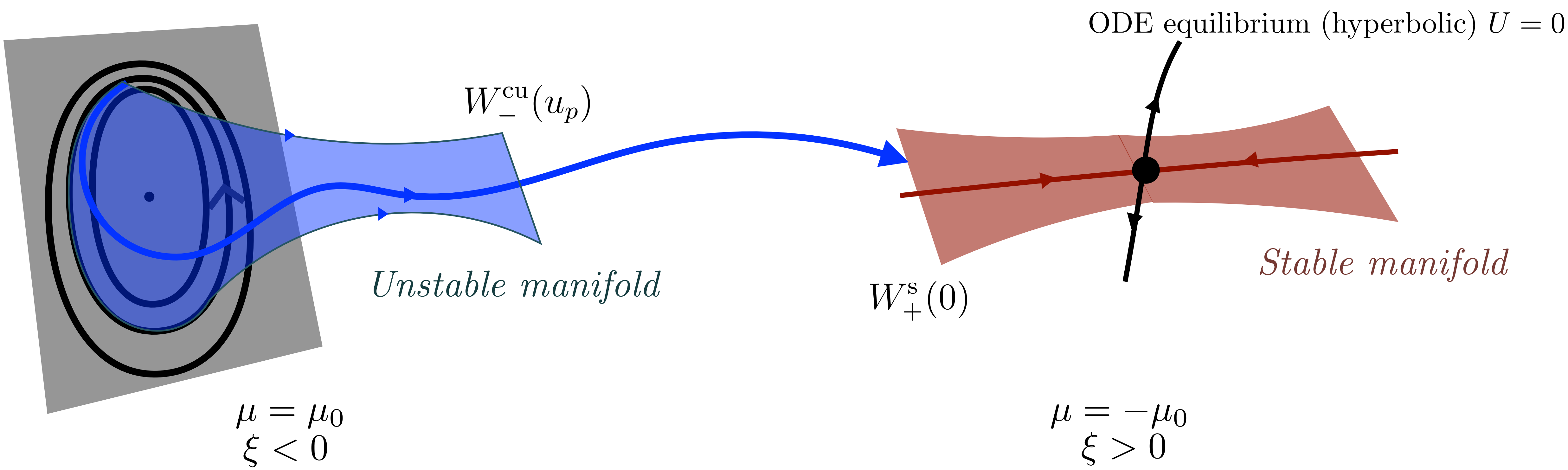
- Look for front solutions as heteroclinic orbits:
- Intersections of *invariant manifolds* $W_{-}^{\mathrm{cu}}(u_p) \cap W_{+}^{\mathrm{s}}(0)$ of asymptotic states
  - $W^{\mathrm{s/u}}(0) :=$ *set of trajectories which converge to state in forwards/backwards evolution*
- **Geometry of intersection gives wavenumber predictions!**

$$
\begin{aligned}
u_\xi &= v,\\
v_\xi &= w,\\
w_\xi &= \theta,\\
\theta_\xi &= (\mu(\xi) - \omega\partial_\tau)u + cv - w - u^3
\end{aligned}
$$

stripe patterns $u_p(k_x\xi + t)$

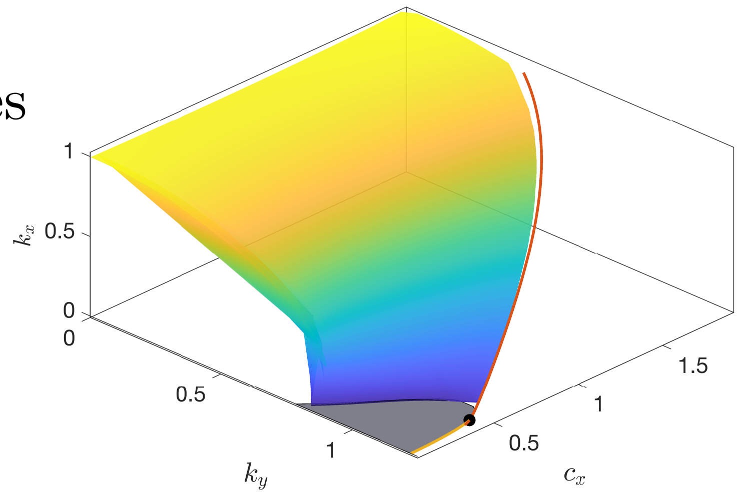


periodic orbits



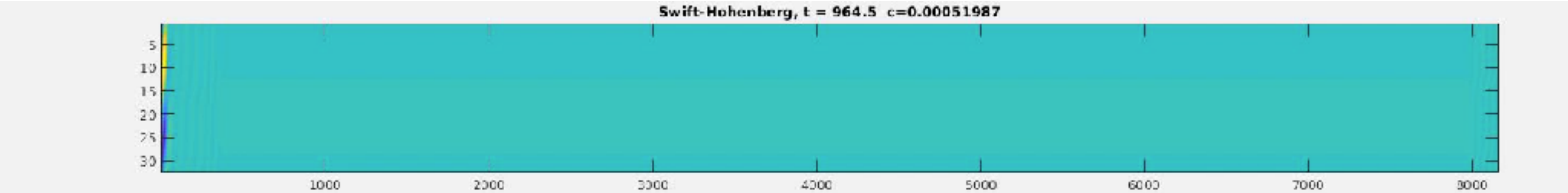
Exploring 2-dimensional patterns

- How to characterize relationship between patterns and growth speed?
- Rigorous analysis requires application/development of new techniques
- Explore (k_y, c)- parameter space first using direct simulations
- Fix a vertical period k_y:

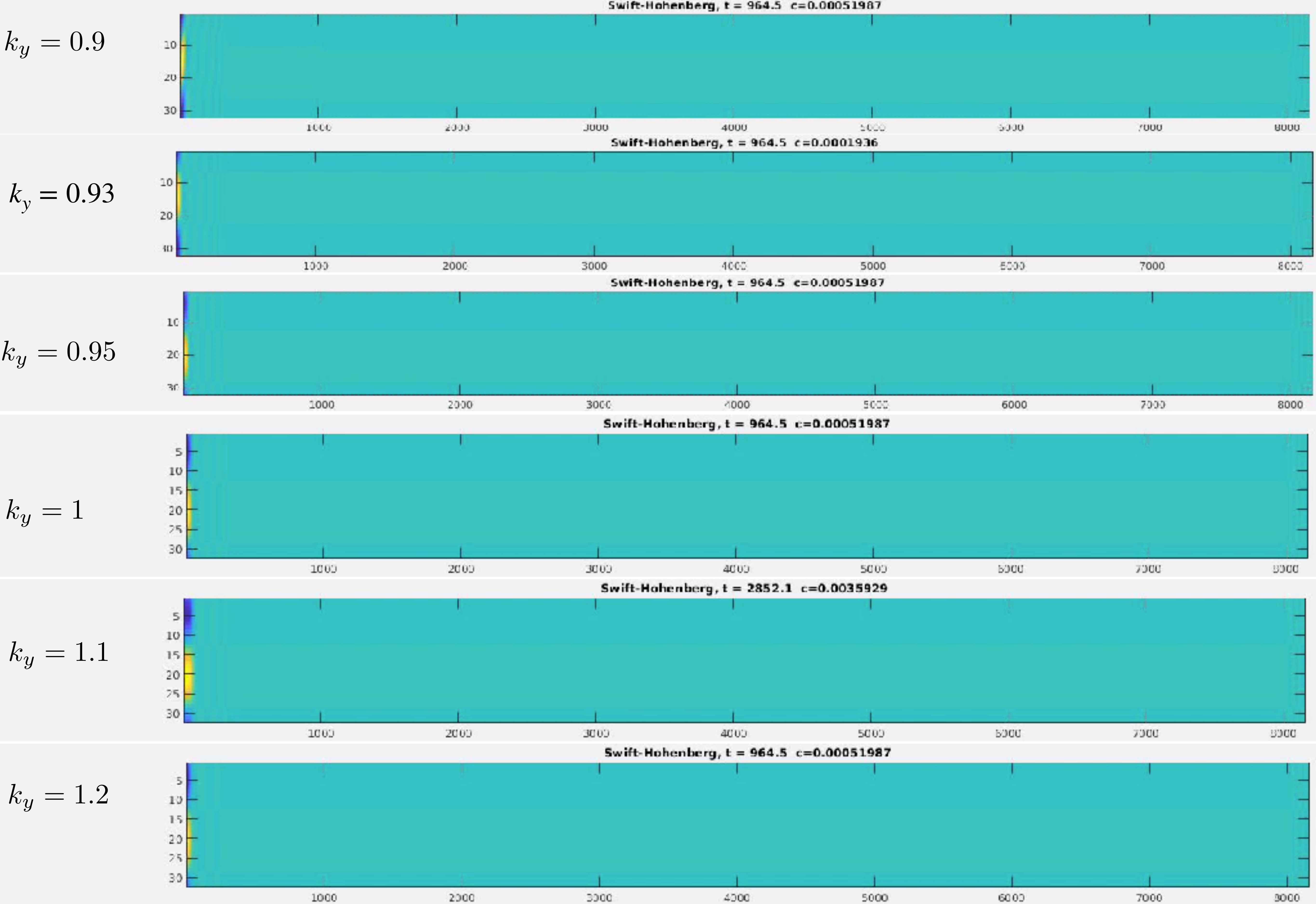


$$\omega u_\tau = -(1+\partial_\xi^2 + k_y^2\partial_y^2)^2 + \mu u - u^3 + c\partial_\xi u \qquad y \in [0,2\pi/k_y)$$

- Exponentially growing quenching/growth front $\mu(x-\zeta(t)t), \quad \zeta(t) \sim e^{\epsilon t}$
- Freeze pattern in the wake (gets rid of possible secondary instabilities)



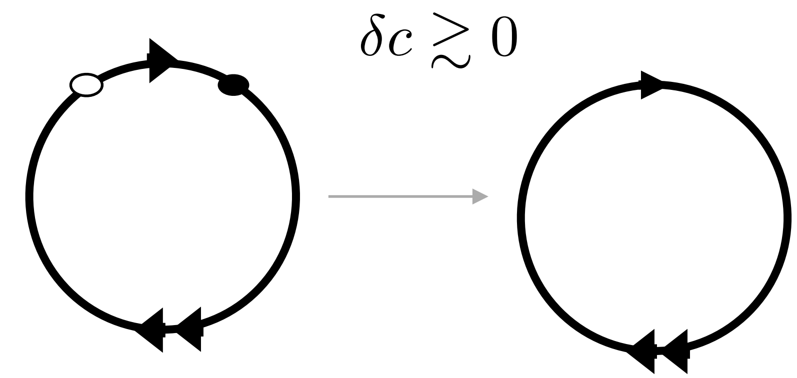
$k_y = 0.85$



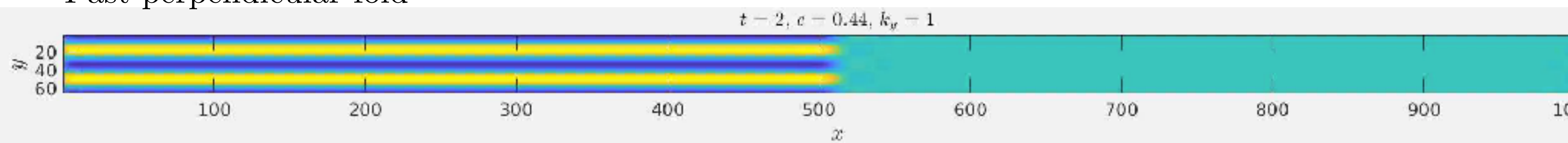
$k_y = 0.9$
Swift-Hohenberg, t = 964.5 c=0.00051987
$k_y = 0.93$
Swift-Hohenberg, t = 964.5 c=0.0001936
$k_y = 0.95$
Swift-Hohenberg, t = 964.5 c=0.00051987
$k_y = 1$
Swift-Hohenberg, t = 964.5 c=0.00051987
$k_y = 1.1$
Swift-Hohenberg, t = 2852.1 c=0.0035929
$k_y = 1.2$
Swift-Hohenberg, t = 964.5 c=0.00051987

Oscillatory behavior past saddle-nodes

- Take c just above one of the saddle-node values
- Phase sees the “ghost” of the heteroclinic solution of MTW:
- Dynamics similar to saddle-node on a limit cycle:
  - Period of oscillation scaling like $\sim (\delta c)^{-1/2}$

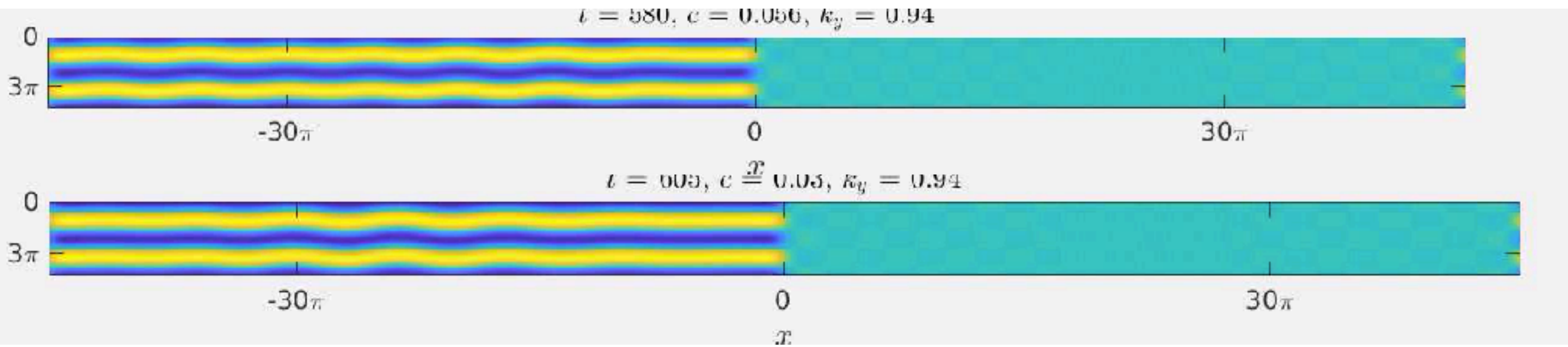


Past perpendicular fold



“Phase-kink” shedding

Past oblique fold

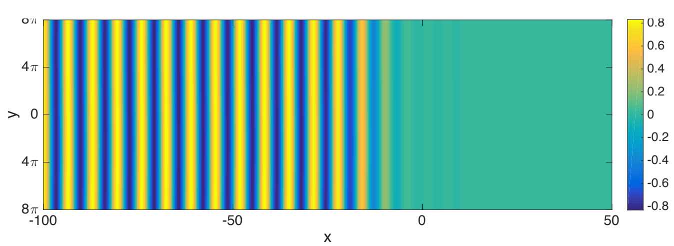


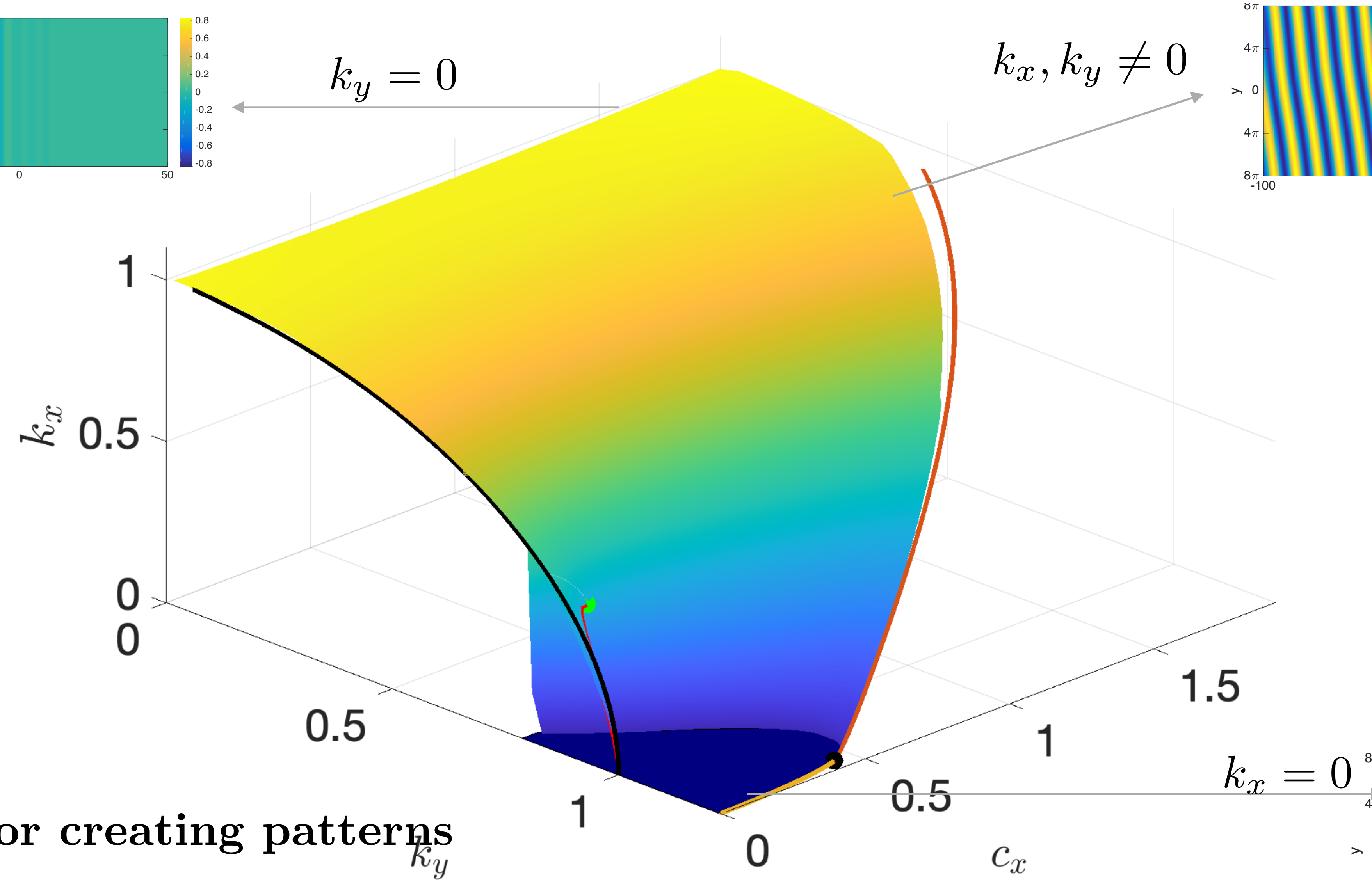
“Wrinkled” patterns

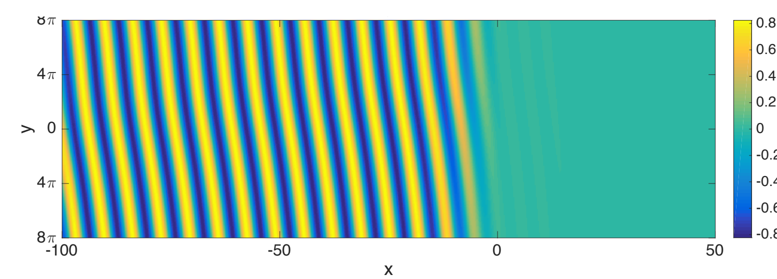
Organize/represent solutions: "Moduli space"

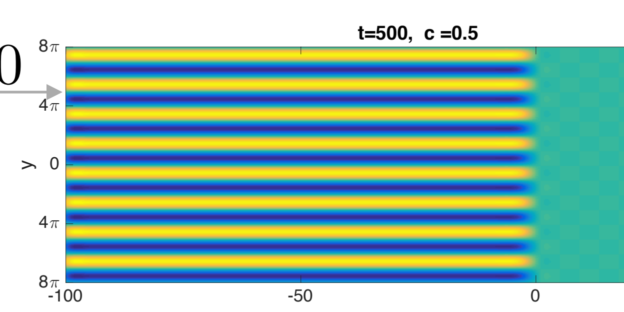
$$\text{MTW} \quad \begin{cases} \omega\partial_\tau u = -(1+\partial_\xi^2 + k_y^2\partial_\tau^2)^2 u + \mu(\xi)u - u^3 + c\partial_\xi u, & u(\cdot,\tau)=u(\cdot,\tau+2\pi) \\ \lim_{\xi\to-\infty} u(\xi,\tau) \to u_{\mathrm{p}}(k_x\xi+\tau,k), \quad \lim_{\xi\to\infty} u(\xi,\tau)\to 0, & \mathbf{k}=(k_x,k_y), \omega = ck_x \end{cases}$$

$\mathscr{M} := \{(k_y, c, k_x) \in \mathbb{R}^3 \ : \ \text{MTW has a solution}\}$—>Each point on surface represents a striped pattern



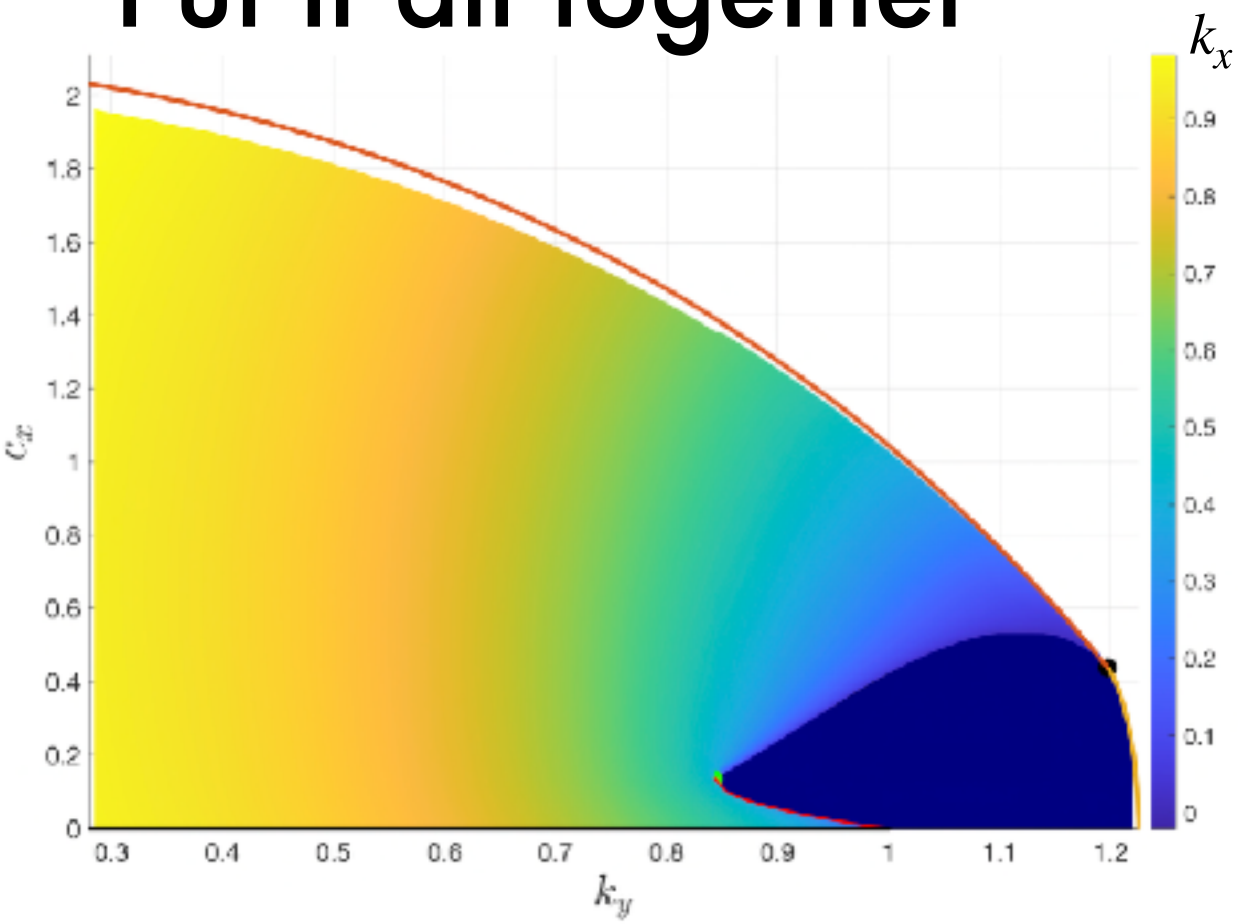


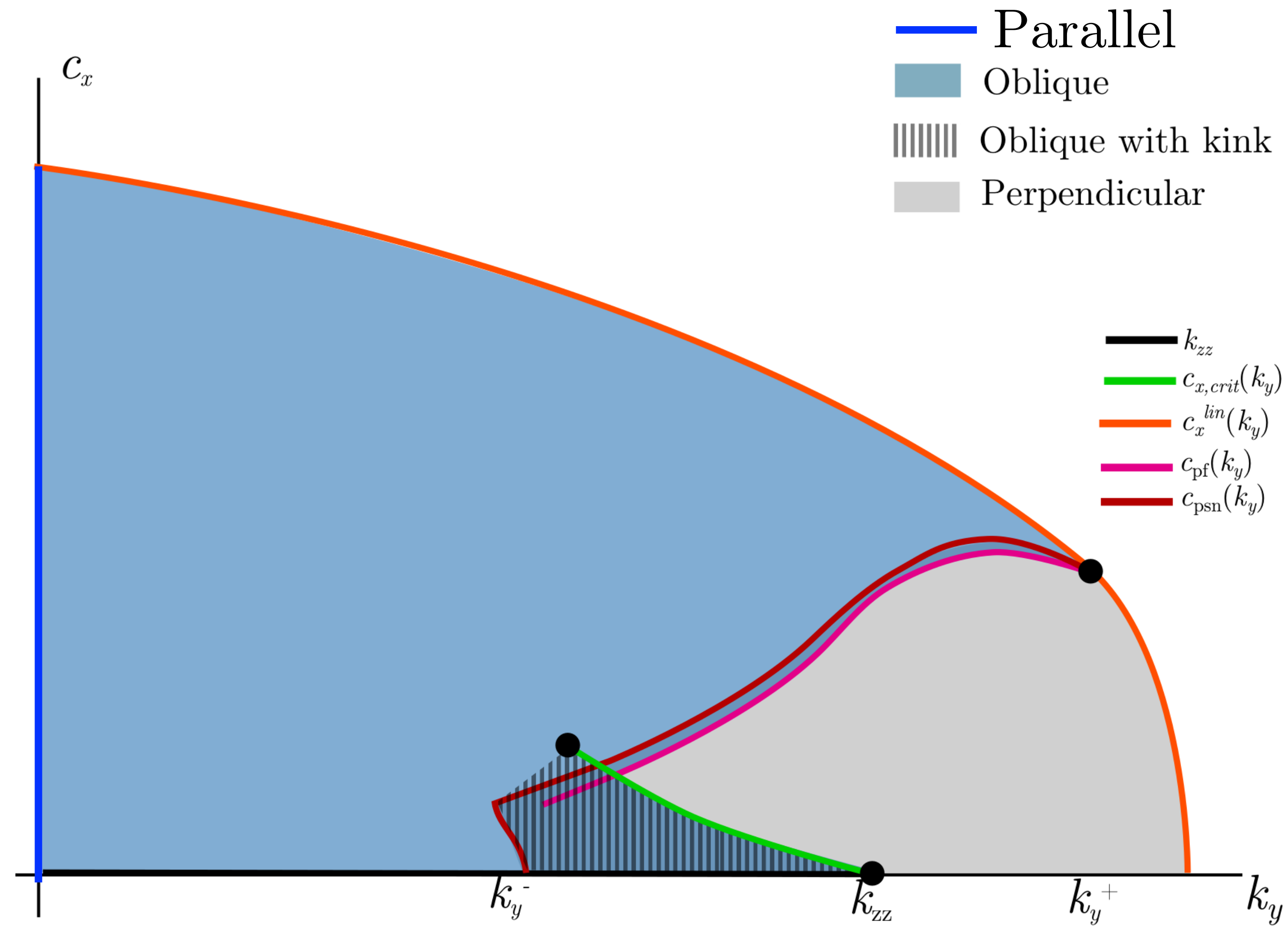




"Cookbook" for creating patterns

Put it all together

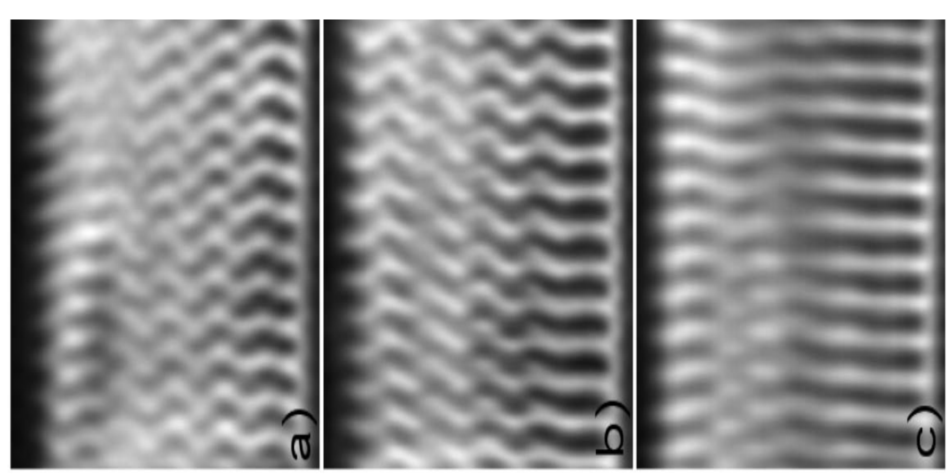


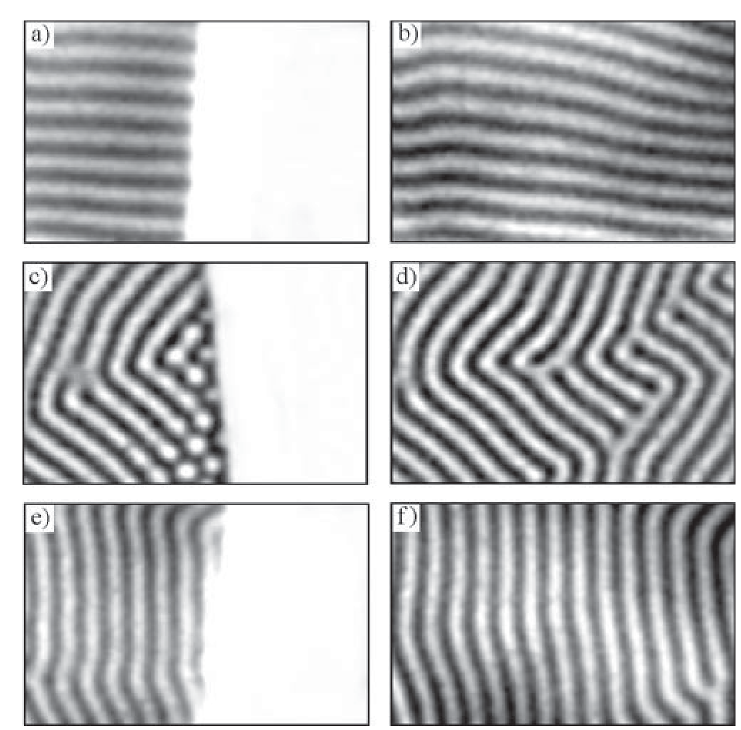


Boundaries of pattern transitions governed by bifurcation curves

Moduli space $\mathscr{M}$ is the "pattern cookbook":

how to create and select a given pattern

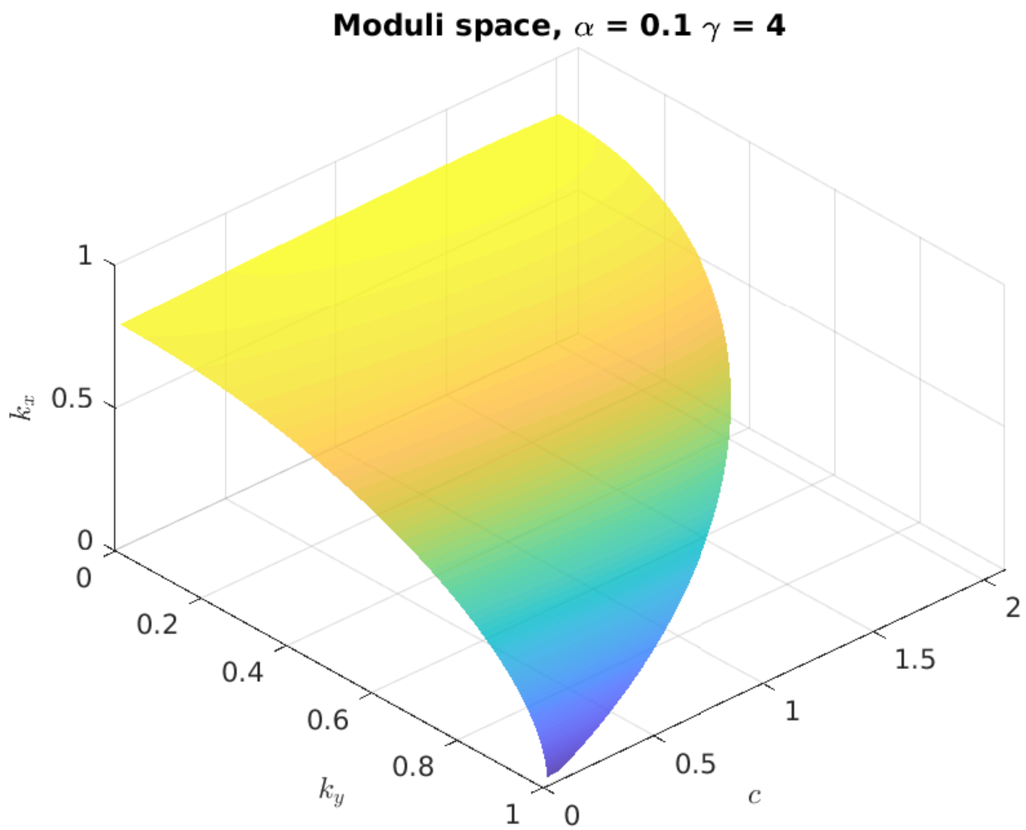




Other systems: how do patterns behave?

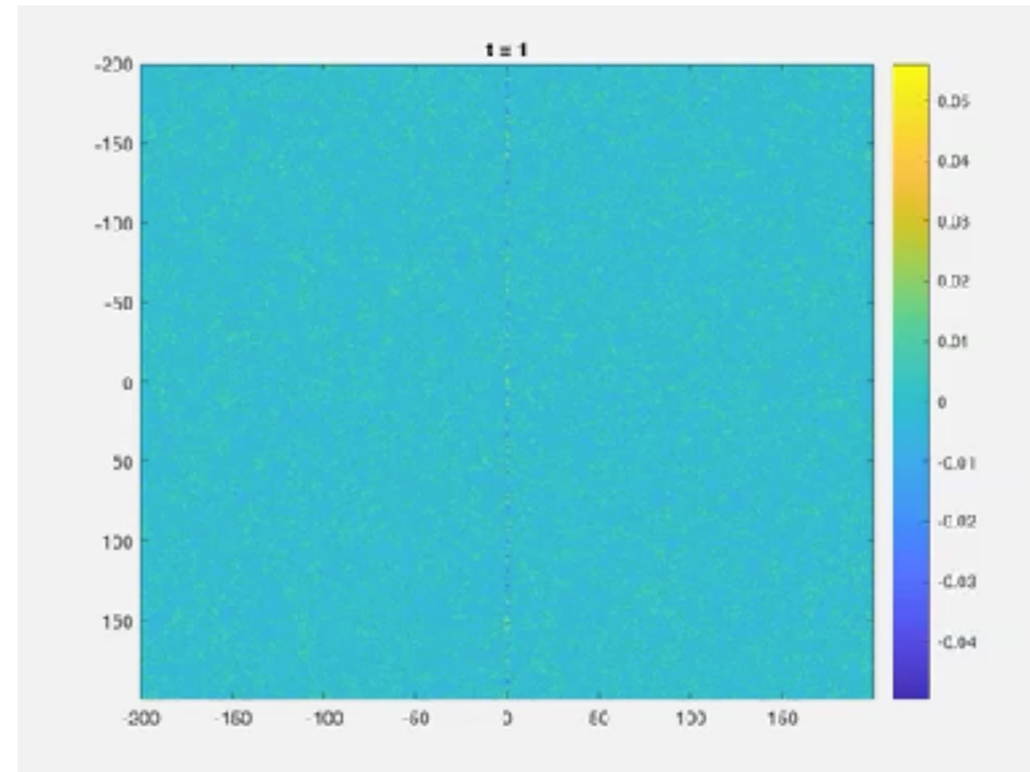
Complex Ginzburg Landau equation

$$A_t = (1 + \mathrm{i}\alpha)(\partial_x^2 + \partial_y^2)A + \mu(x - ct)A - (1 + \mathrm{i}\gamma)A|A|, \quad A \in \mathbb{C}$$



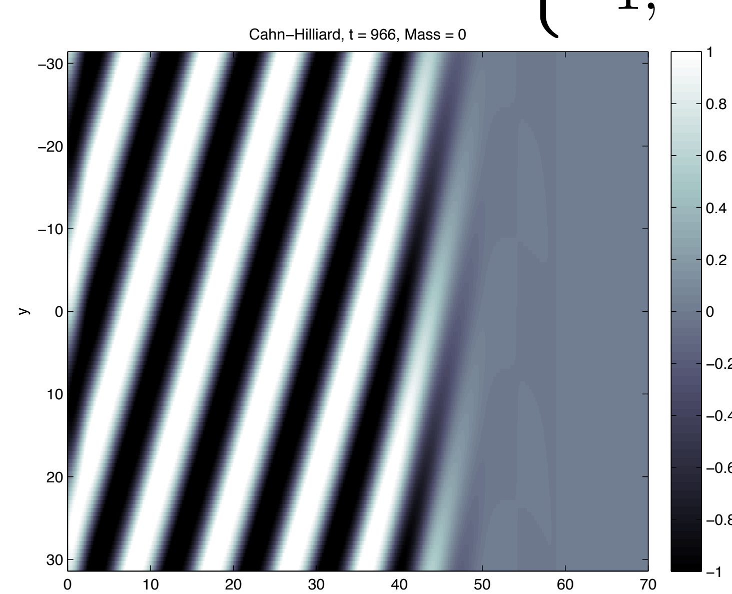
Reaction-Diffusion systems

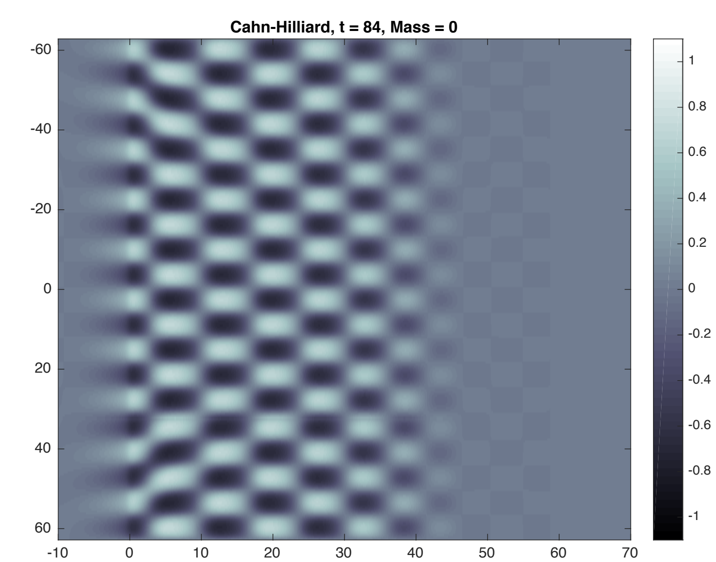
$$u_t = d_u u_{xx} + \mu(x - ct)u - u^3 - v$$
$$v_t = d_v u_{xx} + u - \gamma v$$



Cahn-Hilliard: (phase separative systems) [RG, Scheel '15]

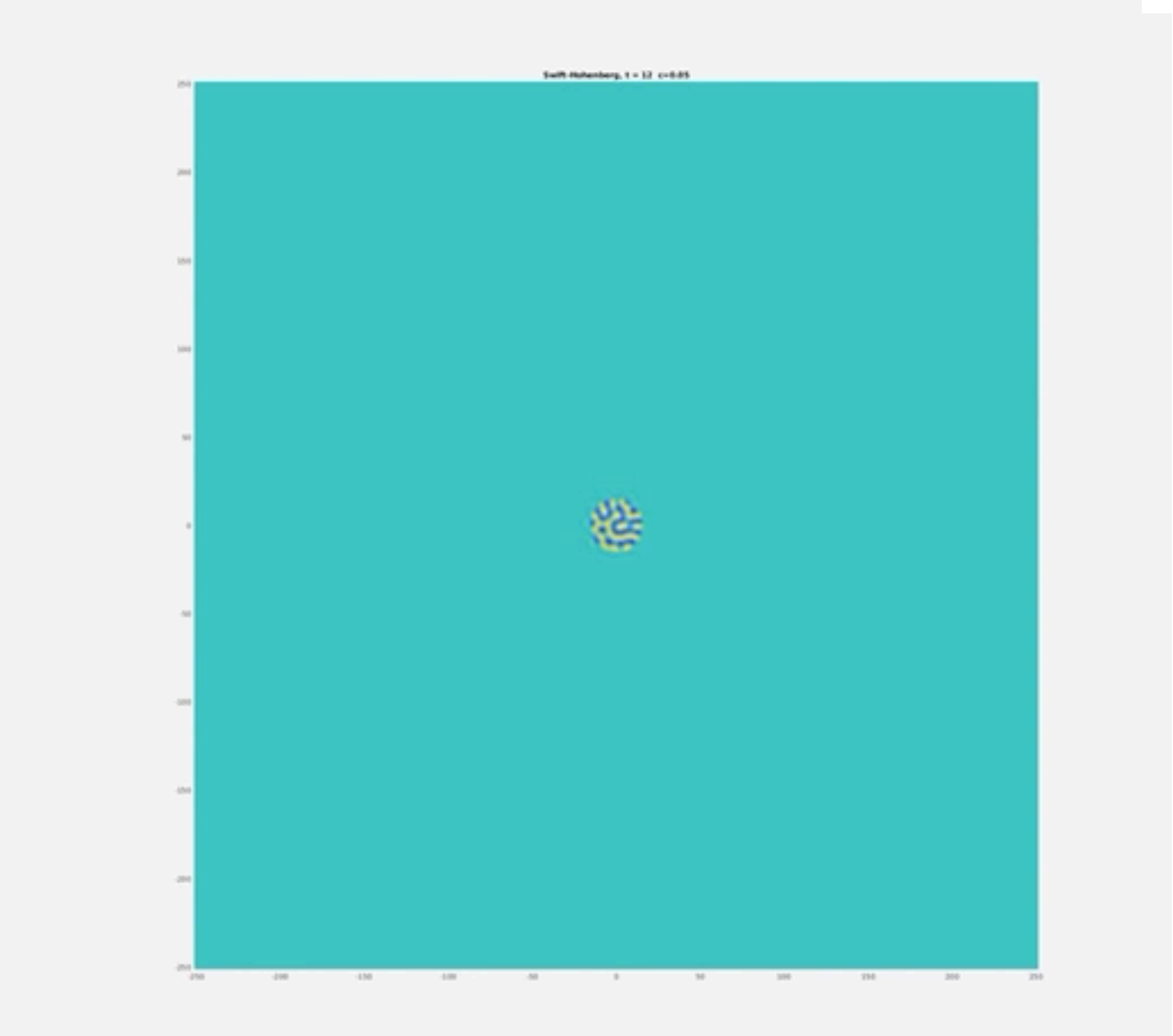
$$u_t = -\Delta(\Delta u + \chi(\xi)u - u^3) + cu_x, \qquad \chi(\xi) = \begin{cases} 1, & \xi \in [-L, L] \\ -1, & \xi \in [-\infty, -L] \cup [L, \infty] \end{cases}$$





Other types of growth

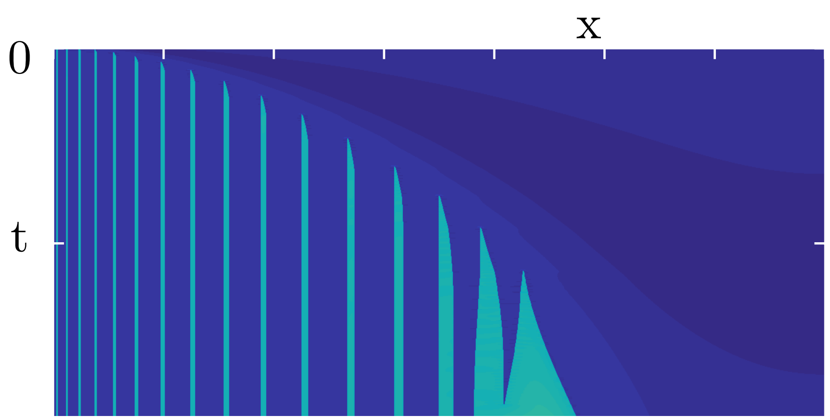
- For example: radial growth front

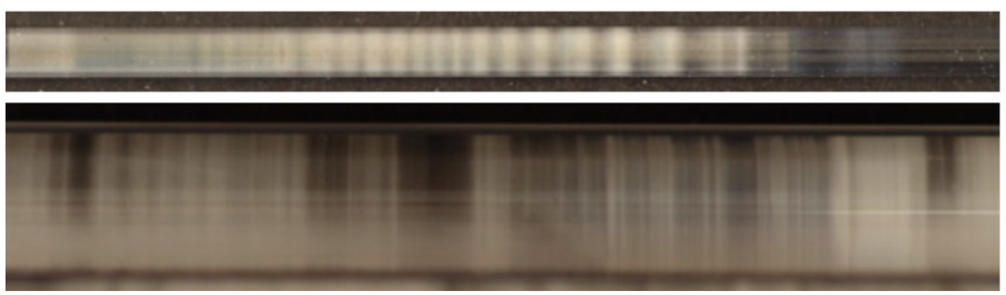


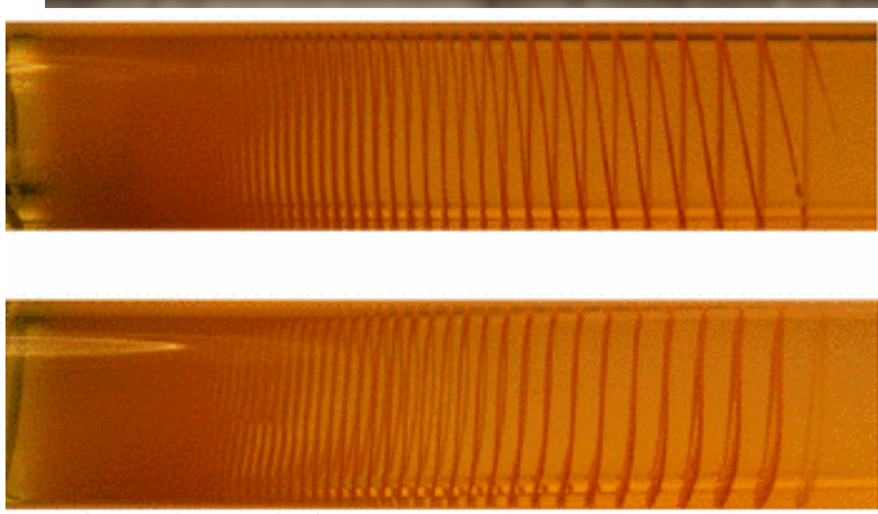
Patterns in experiment

Precipitation and Vapor deposition

$$a_t = D\Delta a - f(a,b) + h(t,x;c)$$
$$b_t = \Delta b + f(a,b)$$

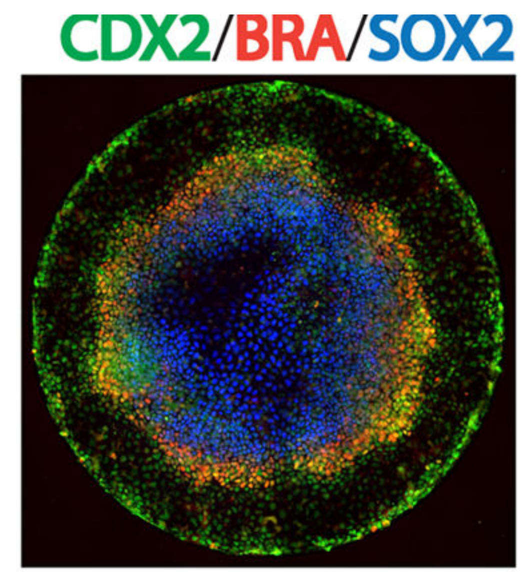


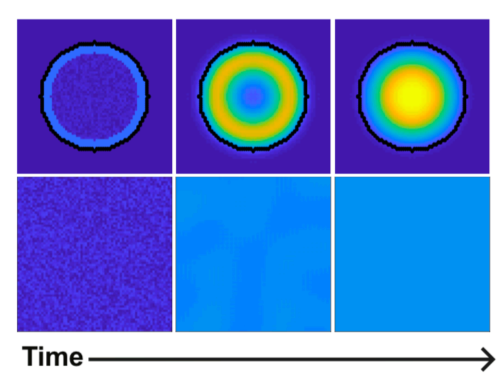


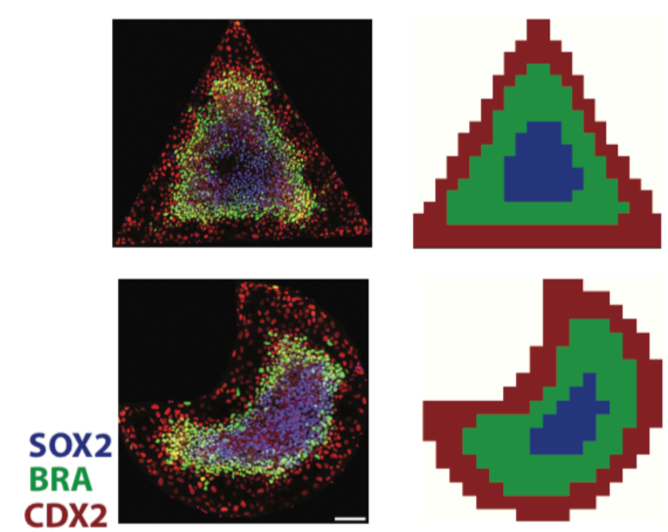


Gastrulation in embryos

$$A_t = D_A \Delta A + \frac{s_A A^2}{k_I + I)} - k_A A$$
$$I_t = D_I \Delta I + s_I A^2 - k_I$$







w/ P. Shipman, S. Thompson

[S. Chhabra, L. Liu, RG, A. Warmflash,'19]

Summary

- Nature is incredibly capable of forming patterns and structure
- Growth is a useful way to mediate pattern formation in natural and experimental settings
- Mathematics can characterize the phenomena:
  - Dynamics and Functional Analysis are powerful viewpoint to illuminate the underlying structure/mechanisms in PDE models
  - 1-D patterns: existence and wavenumber selection for fast and slow growth speeds
  - 2-D patterns: Transitions between different orientations of stripes, many interesting dynamics and phenomena!
  - Use the moduli space representation as a *pattern cookbook*
  - Yields explicit qualitative/quantitative predictions for pattern selection
- *Many of these predictions can be used in other PDE models*

- There is much more to be done, using a variety of tools and approaches:
  - **Rigorous approaches, formal asymptotics, numerics**. . .

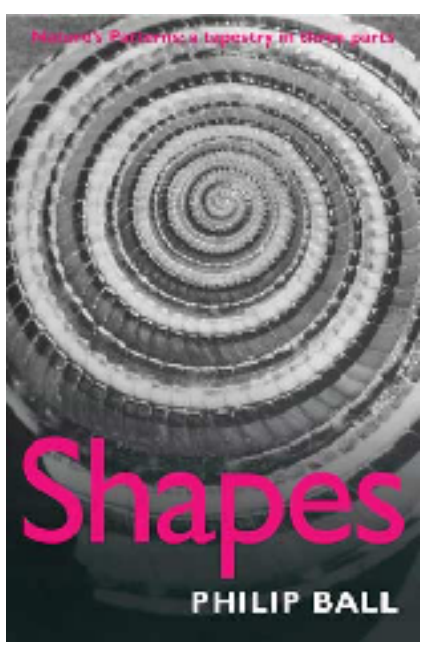
Some good references to get into this area

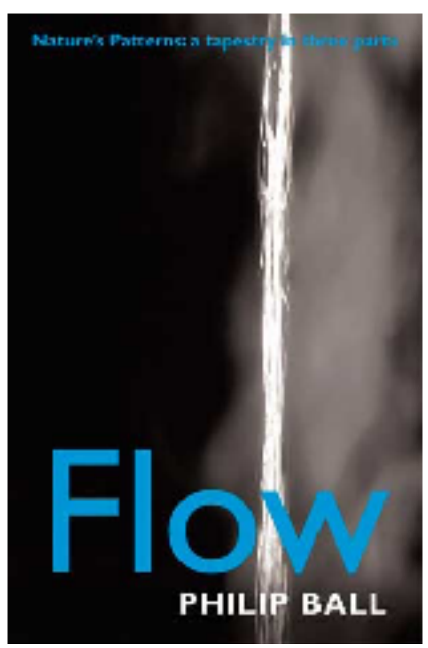
- *"Forging patterns and making waves from biology to geology: a commentary on Turing (1952) 'The chemical basis of morphogenesis'*, Ball, Phillip, *Phil. Trans. R. Soc. B 370: 20140218.*

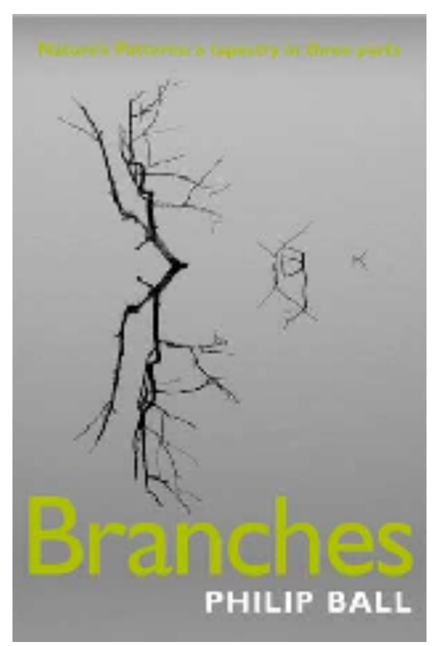
- *The Chemical Basis of Morphogenesis A. M. Turing, Phil. Trans. R. Soc. B 237: No. 641. (Aug. 14, 1952), pp. 37-72*

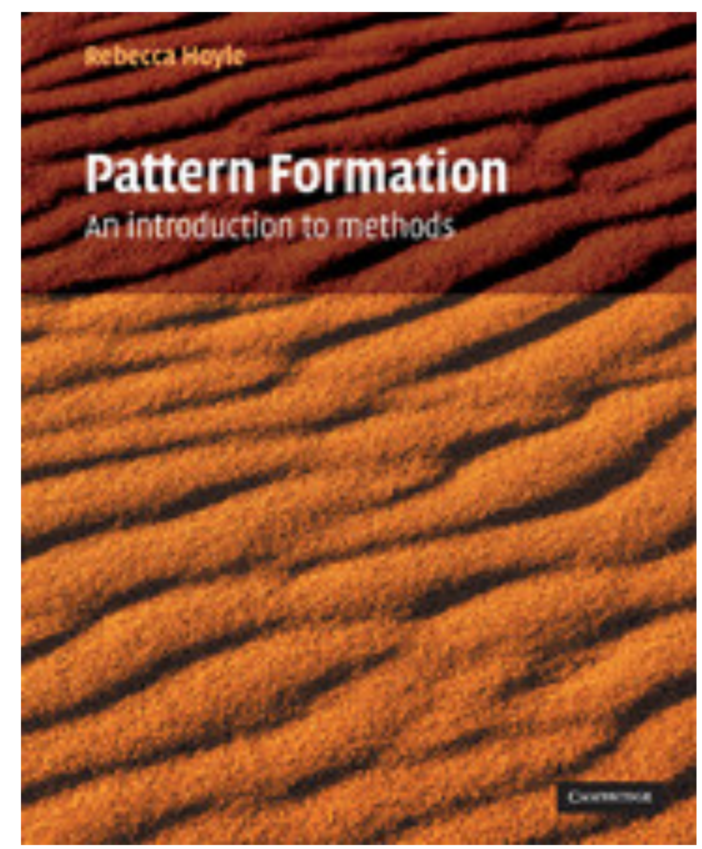
- *Nature's Patterns: a tapestry in three parts*, Phillip Ball

- *Pattern Formation: An Introduction to methods*, Rebecca Hoyle



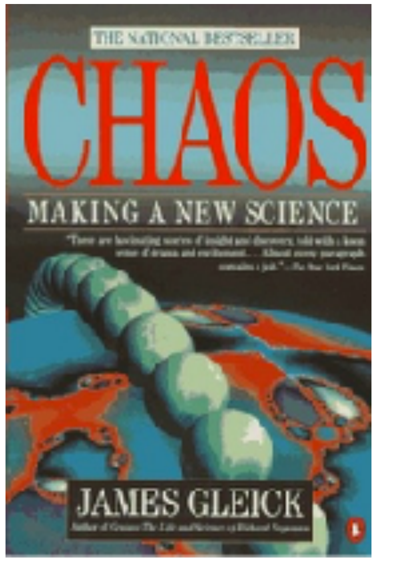






Career as an academic mathematician

My path



- Always liked math, was decent at it (mom was a high-school math teacher)
- Started reading non-technical books on math (non-Euclidean geometry, Riemann Hypothesis, Gödel's incompleteness) one stuck out:
  - *Chaos*, by James Gleick
- (2007-2011) Attended Michigan State University: B.S. in math, B.A. in physics
  - Research in dynamics of piecewise-linear maps, and ODE modeling of dye-sensitized solar cells
  - Attended an REU at Univ. of Minnesota with Arnd Scheel
- (2011-2016) PhD in Mathematics at University of Minnesota: studied dynamical systems, functional analysis, partial differential equations, with applications to formation of coherent structure in nature
- (2016-2019) Postdoctoral fellowship at Boston University, mentored by Prof. C. Eugene Wayne
- (2019-) Tenure-track assistant professor at Boston University

Undergrad studies in math/applied math

- Explore!!!
  - Take classes, go to talks, meet with faculty
  - Directed/independent study, research project with faculty,
  - Summer REU/Interships (academic vrs. industry career paths)
  - Maybe teach a little (?), grading, teaching assistant, etc…
- Start thinking about graduate school
    - How to prepare: all the above!, discuss with faculty advisors, start reading and thinking about types of research
    - Choosing one: don't just go on rankings
      - Want a school with at least at least few research areas/faculty that interest you
      - What is the grad student culture/community like?
      - Where do PhD graduates go after?
      - Location and benefits?

Graduate School

- Last place where you just get to learn! (and learn how to learn, establish habits)
- It's hard, but mostly fun! (good to have a support group)
- Learn mathematics more deeply (core subjects algebra, topology, analysis, applied math)
- Learn one or two subjects really, really well
- After introductory course work, start reading papers with faculty, start a small project,
- Typically have to pass written/oral exams
- Stipend support by either Teaching Assistantship, Research support from advisor
- Math is social!   (i.e. soft-skills matter too!)
  - Talk with professors (possible collaborations, will need letters)
  - -> Go to workshops/conferences/summer schools, present a poster, give a talk, maybe even collaborate with someone!
  - Organize department events (SIAM, MAA, AMS, AWM student chapters)
- Maybe do internship? (Math PhD's can go into industry!)
- Start thinking about career track: Research University (large/small, public/private), liberal arts 4-year, national lab.

Postdoctoral studies

- Become an independent researcher - though typically mentored by a senior faculty
- Move into different research areas
- Gain experience as a lecture/instructor of record (teach various courses, 1 to 2 (maybe 3) a semester)
- Start applying for tenure-track jobs
- Taking on more responsibilities:
  - Mentor undergrad research
  - Organize professional events, referee journal papers
  - Maybe work of multiple projects
  - Help with department functions (write prelims, organize department seminars, . . .)

Tenure- Track Assistant Professorship

- Research:
  - develop and produce high-quality, impactful research (papers, review articles, etc…)
  - Maybe recruit a graduate student or two
  - Be active in your research community
- Teaching: (one to two courses/a semester, varies depending on institution)
  - high-quality instruction (student evaluations and peer-reviews)
  - Variety of courses (large 100-200 level lectures, 500-advanced undergrad/masters classes, graduate courses)
  - Maybe develop a new course or two?!
- Service:
  - Department: take part in administration and direction of department/school
  - University: faculty council, etc…
  - Community: academic and public

Academic Career: Pros and Cons

Pros:

- Get to do math for a living!
- Relatively independent (still have bosses, but less direction than at a company)
  - Academic freedom and tenure
- No profit incentive (though have to get grants!)
- Relatively flexible schedule
- Get to visit a lot of cool places and meet interesting and diverse people
- Contribute new knowledge to the world
- Educate/Impact the next generation of mathematicians and scientists
- Sabbaticals are nice
- Job security (once you get one…)

Cons:

- Positions are competitive (difficult to get)
- Pay not at the level of industrial job with equivalent experience (though not bad at all!)
- Work/Life balance can be tough (especially during early career)
- Societal/economic trends & broad changes in academia (student debt bubble, etc…)
- Need to bring in grant $$ for university
- “Publish or perish”

Day in the life (on “teaching day”)

- 5:30-6am wake-up, breakfast, get ready, bike in around 7am
- 7:15-7:30am - Arrive on campus, respond to emails
- 7:30 - 9: work on a research problem
- 9-10: teaching prep, review lecture notes, grading, course emails
- 10-11: teach
- 11-12 decompress & send emails (maybe lunch)
- Afternoon (varies):
    - Research collaboration meetings, work on research projects
    - Office hours, student research projects
    - Committee/faculty meetings (undergrad, graduate, etc. . .)
    - Referee journal articles
    - Other activities for more senior faculty (advising, university committees, editor of academic journals etc. . .
    - Go to seminar talks
- 4 - 6pm: Bike home
- 6-8:30pm: Dinner and family time, 8:30-10pm work on a research project, 10-10:30 get ready for bed

Thanks!!

Thanks!

References:

- Avery, RG, Goodloe, Milewski, Scheel, Growing stripes, with and without wrinkles, in review, (2018).

- S. Chhabra, L. Liu, RG, A. Warmflash. Dissecting the dynamics of signaling events in the BMP, WNT, and NODAL cascade during self-organized fate patterning in human gastruloids. , bioRxiv 440164;

- RG, Scheel, A. *Pattern-forming fronts in a Swift-Hohenberg equation with directional quenching — parallel and oblique stripes*, J. Lon. Math. Soc., (2018).

- RG, Scheel, A., et. al. *Universal wavenumber selection laws in apical growth*, Phys. Rev. E, 2016

- RG, Scheel, A, *Hopf bifurcation from fronts in the Cahn-Hilliard equation, Arch. Rat. Mech. Anal., 2015*

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- **RG would like to acknowledge the partial support of grant NSF-DMS-1603416**