MA561 - Fall 2024 Homework 4 - Due Friday, Oct. 11th, 9am on Gradescope

Note, for all problems which ask you to graph the solution, you are welcome to embed figures made from a software package such as Mathematica or Matlab to more efficiently and accurately plot solution profiles you obtain. For the Fourier series questions, I encourage you to do as many calculations by hand as you can, but please feel free to check your work and plot approximations using mathematica or another software.

Problem 1 *(Olver 3.1.2(b)):* Find all separable eigensolutions to the heat equation $u_t = u_{xx}$ on the interval $x \in [0, \pi]$ subject to mixed boundary conditions $u(0,t) = 0,\ u_x(\pi, t) = 0$.

Problem 2 *(Olver 3.1.5):*

(a) Find the real eigensolutions to the damped heat equation $u_t = u_{xx} - u$.

(b) Which solutions satisfy the periodic boundary conditions $u(-\pi, t) = u(\pi, t),\ u_x(-\pi, t) = u_x(\pi, t)$?

Problem 3 *(Olver 3.1.6):* Find the real eigensolutions to the diffusive transport equation $u_t + cu_x = u_{xx}$. modeling the combined diffusion and transport of a solute in a uniform flow with constant wave speed c.

Problem 4 *(Olver 3.2.1 (h)):* Find the Fourier series of the function $x \cos x$.

Problem 5 *(Olver 3.2.25):*

(a) Sketch the 2π-periodic half-wave $f(x) = \begin{cases} \sin x, & 0 < x \leq \pi \\ 0, & -\pi \leq x < 0 \end{cases}$

(b) Find its Fourier series

(c) Graph the first five Fourier partial sums and compare with the function $f(x)$

(d) Discuss convergence of the Fourier series (hint: might be helpful to plot $|f(x) - s_n(x)|$ where s_n is the n-th Fourier series.)

Problem 6 *(Olver 3.2.51 + 3.3.1):* Consider the ramp function $\rho(x) = \begin{cases} x, & 0 < x \leq \pi \\ 0, & -\pi \leq x < 0 \end{cases}$.

(a) Use the Fourier series for the step function (Eqn. 3.49 in the text), and integration, to find the Fourier series for ρ (explaining why you can do this).

(b) Then, find the Fourier series for the second-order ramp function $\rho_2(x) = \begin{cases} \frac{x^2}{2}, & 0 < x \leq \pi \\ 0, & -\pi \leq x < 0 \end{cases}$