MA561 - Fall 2024 Homework 6 - Due Friday, Nov. 8, 5pm on Gradescope

Problem 1 *Olver 4.2.9*: Let $a, c > 0$ be constants. The telegrapher's equation $u_{tt} + au_t = c^2 u_{xx}$ represents a damped version of the wave equation. Consider the Dirichlet boundary value problem $u(0,t) = 0 = u(1,t)$ on the interval $x \in [0,1]$ with initial condition $u(x,0) = f(x),\ u_t(x,0) = 0$.

(a): Find all separable solutions to the telegrapher's equation that satisfy the boundary conditions.

(b): Write down a series solution for the initial boundary value problem.

(c): Discuss the long term behavior of your solution.

(d): State a criterion that distinguishes overdamped frokm underdamped versions of the equation.

Problem 2 *Olver 4.2.19:* Consider the wave equation $u_{tt} = u_{xx}$ on the interval $0 \leq x \leq 1$ with homogeneous Dirichlet boundary conditions at both ends.

(a) Use the d'Alembert formula to explicitly solve the initial value problem $u(x,0) = x - x^2, u_t(x,0) = 0$.

(b) Graph the solution profile at some representative times, and discuss what you observe.

(c) Find the Fourier series of the form the solution and compare with that of part (a).

(d) How many terms do you need to sum to obtain a reasonable approximation to the exact solution?

Problem 3 *Olver 4.3.10(b):* Solve the boundary value problem for Laplace's equation on the square $\Omega = \{0 \leq x \leq \pi,\ \ 0 \leq y \leq \pi\}$ with $u(x,0) = 0, \qquad u(x,\pi) = 0, \qquad u(0,y) = \sin(y), \qquad u(\pi,y) = 0.$

Problem 4 *Olver 4.3.27:* (a) Find the equilibrium temperature on a disk of radius 1 (i.e. Laplace's equation on the unit disk), when half the boundary is held at 1 degree and the other half is held at -1 degree.

(b) Find the equilibrium temperature on a half-disk of radius 1 when the temperature is held to 1 degree on the curved edge and 0 degrees on the straight edge.